



Design, Simulation, and Fabrication of Bias-Curvature Electromagnets for Cold Atom Experiments

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Physical Sciences**

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Abstract

Quantum gas experiments are an established tool in atomic physics to create quantum analogues for electronic systems with exotic quantum behavior. These systems are important to study the atomic physics of quantum systems which simulate various the electronic systems. This simulation then illuminates the inner workings and behaviours of quantum computation or electronic technology. Electromagnet construction is an important component of these experiments as they are necessary for, magneto-optical trapping of atoms, and changing different interaction parameters that govern cold atom behaviour. Through the use of bitter-type electromagnets, we aim to create what we call Bias-Curvature coils: a Bitter-type electromagnet design for the quantum simulation using lithium-6 atoms, specifically designed to create fields above 832 gauss, the Feshbach resonance of lithium-6 fermions, but also able to ensure that the field has a controllable inhomogeneity—even up to zero magnetic curvature. In doing so, we unlock the potential for a new quantum gas experiment that potentially explores near quantum critical points for quantum matter. We use a finite element methods simulation in FreeFEM++ to computationally solve for the weak solution to Maxwell’s equations. This produced a finite element estimation of the bias-curvature coil’s magnetic field. We used the results to determined design suggestions in constructing such a magnet that would help achieve its expected magnetic output. Lastly, we detail the building and testing of an example electromagnet that is capable of producing 832 gauss with a highly tunable magnetic curvature assuming it is properly cooled.

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As my last name suggests, I should extend my gratitude to those who I hold dear for supporting me to this point. First and foremost, I thank my family—Bunda, Ayah, and Kakak. Their contributions to the production of this capstone extends over 22 years, and my thank yous could never adequately appreciate how much they mean to me.

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To my future self, know that you should be proud of this document. Even if you think it wasn't as impactful as you hoped, or the results weren't as strong as anticipated, recognize the endless effort you put in it. I look forward to seeing who you are and what works you produce, no matter its shape, form, or success.

—Zirdi, nerdy with a Z.

Contents

1	Introduction	7
1.1	Ultra cold quantum gas experiments	7
1.1.1	Quantum simulation with magnets	8
1.2	Magnetic homogeneity in quantum simulation	11
2	Magnet Design	15
2.1	The Problem: field strength and inhomogeneity	15
2.2	Why? The search for a quantum critical point	16
2.3	Nullifying the curvature	17
2.4	Bias-Curvature coils geometry	19
2.4.1	Types of electromagnet design	19
2.4.2	Modified Bitter-design	20
2.4.3	The initial design	21
3	Simulation	25
3.1	Weak formulation for magnetostatics	25
3.2	FreeFEM++	27
3.3	Meshing	28
3.4	Levels of approximation	30
3.4.1	Choice of finite element	30
3.4.2	Simplifying the geometry for currents	30
3.5	Computational methodology	31
3.5.1	Variable parameters	33
3.5.2	Using linearity of fields	35
3.5.3	Curvature estimation—Savitzky-Golay Filter	35
3.6	Simulation results	36
3.6.1	Bias-Curvature mean radius simulation	36
3.6.2	Bias-Curvature width simulation	42
3.6.3	Bias-Curvature gradient simulation	44
4	Fabrication	45
4.1	Parts	45
4.2	Water cooling with gaskets and plumbing	45

4.3	Cutting and sanding	47
4.4	Oxidation cleaning	47
4.5	Assembly	49
5	Testing	54
5.1	Field measurements	54
5.2	Constructing a Bias-Curvature coil	59
5.2.1	Validating linearity	59
5.2.2	Constructing an operational profile	59
5.3	Thermal properties	62
5.4	Concluding remarks	63

CHAPTER 1

Introduction

Magnets are used in all ranges of technology—from fridge magnets to confinement systems in fusion reactors. Of course, these magnets produce fields that vary in strengths, uniformity, and spacial direction. Some magnets are naturally occurring while some metals only become magnets, named electromagnets, when a current is run through them. In the field of atomic physics, certain ultra cold quantum gas experiments require very specialized magnets. In order to observe exotic quantum phenomena, we must be able to produce magnetic fields that are more strong, responsive, and precise than most everyday magnets. Before understanding what magnetic fields we require, we must first understand the how magnetic fields matters in ultra cold quantum gas experiments.

1.1 Ultra cold quantum gas experiments

In 2001, Cornell, Ketterle, and Wieman were awarded the Nobel prize in physics for being able to produce a new quantum state of matter, the Bose-Einstein-Condensate (BEC). Through laser cooling techniques and evaporative cooling techniques, Metcalf and Van der Straten were they able to cool down neutral alkali atoms to just a few billionths of a kelvin above absolute zero [1]. This study then allowed the duo to experimentally observe and understand the properties of BECs.

The advent of this achievement proceeded to inspire a new branch of physics—Ultra cold quantum gas experiments.

Due to the laws of quantum mechanics, environmental factors, or other forms of complexity, some quantum systems are difficult to study directly. Though, ultra cold atoms allow physicists to simulate the behaviours of these difficult quantum systems but in a controlled environment, ideal for experimentation. This is known as quantum simulation. Advances in the field of ultra cold atom experiments have

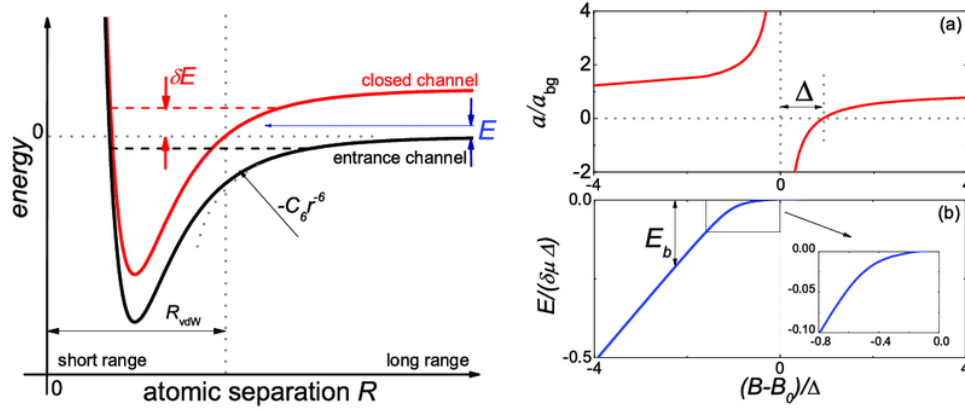


Figure 1.1: A figure representing how the Feshbach resonance arises from the entrance and closed potential channels in the left figure, and how the scattering length diverges near the Feshbach resonance B_0 in the top right figure. The bottom right figure shows the binding energy of a between the atoms as you approach the Feshbach resonance. (reprinted from Chin et al [2] with permission).

led to furthered understanding in high-temperature superconductors, quantum computation, and many other quantum systems relevant to atomic physics.

1.1.1 Quantum simulation with magnets

This paper is concerned with a particular type of quantum simulation that uses magnets to simulate various quantum behaviour. In this section, I elaborate on a selection of ways magnetic fields are used in these quantum simulations.

Feshbach resonances

Let us consider a gas comprised of fermionic atoms, like lithium-6. One of the defining traits of a phase of matter is how the atoms interact with each other. In an ultra cold fermi gas, the atoms get slow enough so that when they collide, they can potentially form a short-lifetime bond to each other known as a Feshbach resonance, one of the interactions that will characterize our quantum phase of matter.

In reference, to figure 1.1, the open channel represents the potential experienced by two atoms freely passing by each other, and the closed channel represents the potential for atoms bound to each other. The Feshbach resonance occurs when the kinetic energy of two atoms colliding in the open channel is equivalent to a stable

bound state energy of the closed channel, denoted δE in the figure. Then the colliding atoms can form this bond through a long range spin interaction. Otherwise, if two atoms don't have the same energy, they'll just remain in the open channel potential and scatter off of each other. It is called a resonance because when the energy of the colliding particles approaches the bound state energy, more atoms meet the requirements to enter this bound state.

Magnetic tuning of Feshbach resonances

It is important to note that the entrance and the closed channels in figure 1.1 have different magnetic moments, which define the energy difference between the two potentials. Because of this, the population of atoms meeting the energy requirement for Feshbach resonance can be controlled by a magnetic field that alters the difference in magnetic moments. Some quantum simulations utilize this feature of the Feshbach resonance to simulate the interaction strengths between interacting atoms in different quantum systems, by varying the magnetic field. The particle interaction strength can then be quantified by a scattering length, which changes according to the magnetic field as follows [3]:

$$a = a_0 \left(1 + \frac{\Delta}{B - B_0} \right)$$

In this equation, a is the resulting scattering length, a_0 is the background scattering length, Δ is the resonance width, B is the external magnetic field applied, and B_0 is the magnetic field where the Feshbach resonance occurs. As seen in figure 1.1, the scattering length diverges when approaching the Feshbach resonance from one direction. Conversely, the scattering length also diverges to negative infinity when approaching from the other. The exact direction depends on the sign of a_0 or Δ . Also, take note of the region where the scattering length goes from positive to negative when passing through $B = B_0 + \Delta$. Crossing this "zero crossing" with a variable magnetic field allows experiments to change from a positive scattering length interaction to a negative one, which leads to interesting quantum behaviours.

BEC-BCS crossover When the Feshbach resonance is magnetically tuned to allow the Fermi atoms to strongly interact, we can observe the two individual atoms form a superfluid comprised of dimers, i.e. two atom bound states that flow with zero viscosity. This quantum behaviour exhibits emergent behaviour that is closely related to a type of quantum matter called

a Bose-Einstein condensate (BEC). When tuned to allow a weaker bond, we can observe two atoms form a superfluid comprised of Cooper-pairs, i.e. atoms that have anticorrelating spin and velocity. This state of matter can be referred to a Bardeen-Cooper Scheiffer (BCS). In between these two states is another interesting quantum phase called the "unitary regime", where the scattering length diverges, and the interaction strength is near the Feshbach resonance. The BEC-BCS crossover is an interesting transition between quantum phases of matter, where we can observe each of these phases on how they transition [4].

Dephasing through single-spin precession

We also consider interaction that governs the behavior of an atom under the presence of a magnetic field. Due to the magnetic moment of particles with spin, atoms in the presence of magnetic field experience a precession with some frequency. There have been quantum simulations to understand these spin-dynamics due to their, again, their analogy to other difficult to study quantum systems.

Many models for quantum simulations consider a constant magnetic field and how it contributes to spin-dynamics. Though, this approximation leads to an oversimplification of the small interacting behaviour of atoms. Thus, it is also interesting to consider how a non-constant magnetic field may change dynamics.

An atom with spin can be subject to magnetic field and then optical confinement, and when using a particular formulation, it is a quantum harmonic oscillator such that their oscillation frequency in the potential is dependent on their spin and local magnetic field [5]. Theory suggests that while a linear magnetic field causes no change in frequency of oscillation, a quadratic magnetic field causes a shift in frequency for the harmonic trap that depends on the atoms spin. This results in relative frequency differences across atoms in the ultra cold gas. In fact, the second derivative, or curvature of the magnetic field, is the leading term that determines the frequency shift. Of course, if a lattice's spins are relatively precessing at different frequencies, interactions and behaviours that depend on the atom's spins will be affected. This feature of quantum simulation is discussed further in a later section

Spins in Bloch sphere representation The spin of a particle can be represented as a vector, named a Bloch vector, of maximum length 1 in a unit sphere. In this representation a Bloch vector point vertically upwards

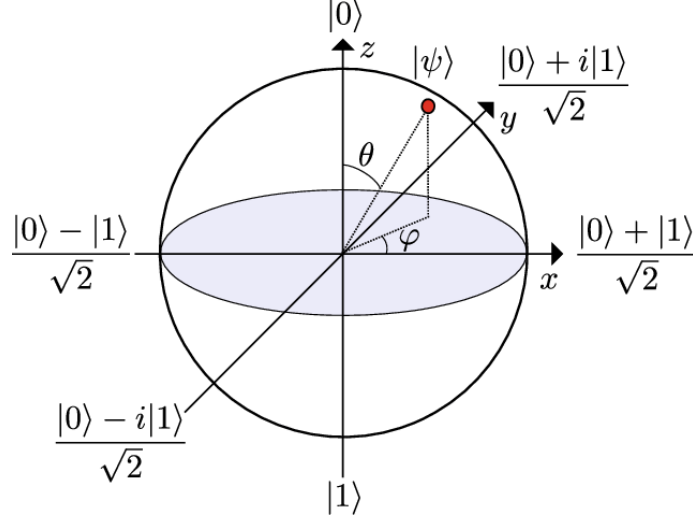


Figure 1.2: The Bloch sphere where an arbitrary spin state $|\psi\rangle$ is defined. Also shown are orthogonal basis vectors like the spin up state $|0\rangle$ and spin down state $|1\rangle$. Other orthogonal basis are shown in the $\hat{x}$ and $\hat{y}$ direction in Bloch representation. (reprinted from Kockum et al [6] with creative commons license by 4.0 permission)

represents spin up and a vector pointing downwards represents spin down, as seen in figure 1.2. The important feature of this representation is visualizing how a particle can exist in a superposition of spins. A Bloch vector pointing in other directions can actually be represented as a linear combination of the two up and down spin states, this is called a superposition. Given the infinite directions that a spin can be defined in Bloch representation, it becomes clear how spins in a sample of atoms can vary greatly.

In the presence of a magnetic field, the spins' magnetic moments will precess about the magnetic field's direction, a phenomena named Larmor precession. This precession causes an oscillation frequency that can be measured using various techniques of spectroscopy. More over, atoms with different spins acting in spin-dependent magnetic potentials will oscillate at different frequencies causing the interesting quantum spin behaviours outlined above.

1.2 Magnetic homogeneity in quantum simulation

In 2019, Smale et al [7], conducted an ultra cold potassium experiment where an external magnetic field was varied across the the zero crossing near the Feshbach

resonance, in order to simulate an out-of-equilibrium phase transition between two dynamical phases of a degenerate fermi gas.

The experiment used a combination of magnetic fields and lasers to create a harmonic potential trap for the potassium atoms. This created a lattice of atoms that could be described with a collective Heisenberg model, a model of magnetism often used for the study of critical points of transition phases of quantum matter.

$$\hat{H}/\hbar = - \sum_i h_i \sigma_i - \sum_{ij} J_{ij} \sigma_i \sigma_j \quad (1.2.1)$$

Here, $\hat{H}$ is the Heisenberg Hamiltonian of the system and is used to calculate the system's time evolution using Schrodinger's equation. σ_i , $i \in \{x, y, z\}$ denotes the X, Y, and Z Pauli operators acting on spins oriented in Bloch space. J_{ij} is a coupling constant that defined interactions across spins in the lattice. Then h_i expresses the strength of an inhomogenous magnetic field for a particular node site in the lattice.

The order parameter, which defines the boundaries for different quantum phases, is the collective magnetization of the spins in the lattice. By measuring the collective magnetization using a Ramsey spectroscopy, the researchers were able to produce a dynamical phase diagram of the quantum fermi gas, with collective magnetization as the order parameter. Figure 1.3 displays that phase diagram that shows the order parameter as a function of inhomogeneity and collective interaction strength. Interestingly, both of these variables are influenced by the magnetic field applied.

Applying a magnetic field, or bias field, results a particular interaction strength for the cold atoms due to the Feshbach resonance. Thus, the magnetic field changes the collective interaction strength of the spins. The summary statistic of these J_{ij} is defined as the thermal average $NJ/2\pi = \langle \sum_{i,j} J_{ij}/N^2 \rangle_T$ in the figure.

Additionally, the magnetic field affects the inhomogeneity parameter h_i , and subsequently, its standard deviation across the node sites $\tilde{h}$.

$$\tilde{h} = \sqrt{\langle \sum_i h_i^2/N - (\sum_i h_i/N)^2 \rangle_T}$$

$$h_i = 2\mathbf{n}_i \cdot \Delta\omega$$

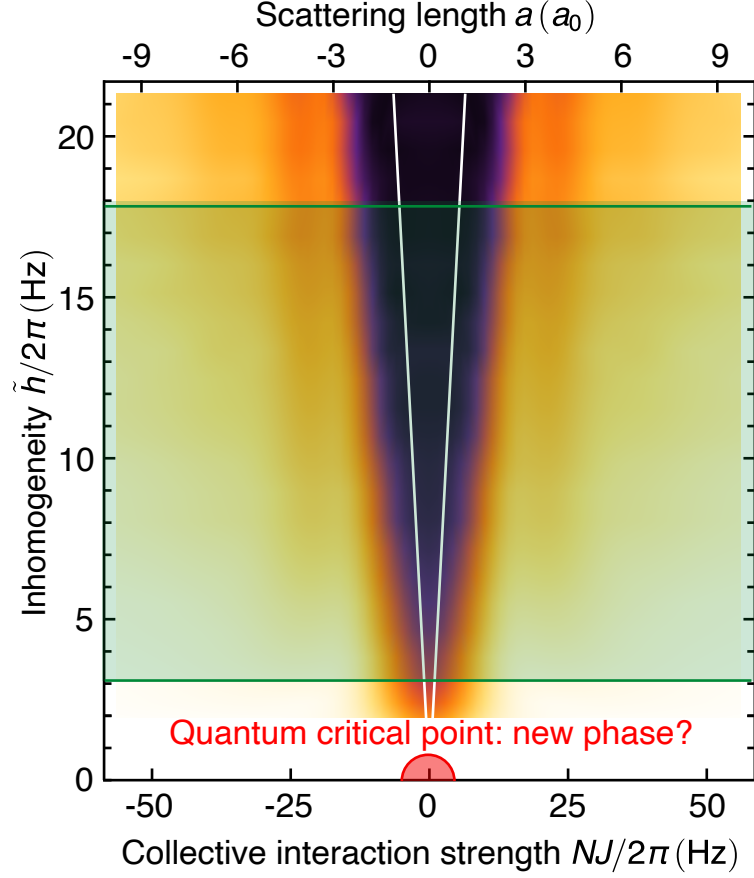


Figure 1.3: Non equilibrium phase diagram with magnetization as the order parameter. The magnetization is denoted with the color, where the dark purple is 0 magnetization and white would be a magnetization of 1. The green region is the domains of the phase diagram that were accessible with the experimental set up from Smale et Al's paper. Note that region for $\tilde{h}/2\pi < 2$ is unmapped with no data.

The atoms in the lattice experience some oscillation frequency due to the harmonic potential trap for the atoms. Consider spin $1/2$ particles sitting in a one dimensional quantum harmonic oscillator trap modeled by a the potential such as $\hat{V}(x) = \frac{1}{2}m\omega^2x^2$. A magnetic field can affect the potential applied on to the particles, and can affect the spin's direction as time evolves. In the presence of a linear magnetic field, the potential experienced by an atom can be modeled with $\hat{V}(x) = \frac{1}{2}m\omega^2(x + \sigma_z x_0)^2 - \frac{1}{2}m\omega^2x_o^2$ [8] where x_0 is a term that shifts, uniformly, the frequency of the spins in either the $+x_0$ or $-x_0$ directions depending on whether the spin is up or down. This would correspond to shifting every h_i in the Heisenberg model as $\Delta\omega$ is constant across all node sights. This constant shift does not change $\tilde{h}$.

Instead, we are more interested in how, the magnetic curvature, or the magnetic field's second derivative, is the leading magnetic contributory to a non-zero h_i . For example, a quadratic inhomogenous magnetic field $B_z(x) = mBx^2/\Delta\mu$ would produce a potential $\hat{V}(x) = \frac{1}{2}m(\omega^2 + \hat{\sigma}_z B)x^2$. Which means that each spin experiences a harmonic frequency $\omega_\sigma = \sqrt{\omega^2 + \sigma B}$ [8], where σ is the eigenvalue of σ_z . Since the frequency is then dependent on the field strength at the node sight, h_i s vary across node sights due to a magnetic curvature, causing a non-zero inhomogeneity $\tilde{h}$. Exploring this regime of spin-dependent potentials has yielded new understandings in spin, motion, Fermi statistics, and interactions for weakly interacting spins in fermionic gases [5].

By mapping out this dynamical phase diagram, Smale et al were able to traverse the regions of inhomogenous spin oscillations, during which, they observed a transitional dynamical phase that was predicted to exist in the Heisenberg model when the inhomogeneity of the atom cloud was fixed to $2\pi \times 18.1Hz$. However, the scope of this experiment is limited by the types of magnetic field they were able to produce. I will discuss this issue further in the next chapter, as it segues into the purpose of this capstone: designing an electromagnet capable of producing magnetic fields that probe new regions of the phase diagram and hence potentially enabling the observation of new quantum phases.

CHAPTER 2

Magnet Design

This capstone’s developments in electromagnet design are pertinent in Yale-NUS College’s Olsen lab for ultra cold quantum gas experiments—where electromagnets are used as to trap, and control cold atoms atoms; Our deliverable magnet enables further research that leads to discoveries in atomic physics as well as wider benefits for the scientific community. Let us delve into how improvements in the magnet design can facilitate our atomic physics experiment.

2.1 The Problem: field strength and inhomogeneity

We return to the collective Heisenberg model [1.2.1] and summarize the character of inhomogeneity in the experiment conducted by Smale et al. The parameter h_i is partially defined by the magnetic field applied onto the lattice of atoms. A combination of optical confinement with lasers, and the external magnetic field applied generates a lattice of potential wells capable of trapping cold-atoms in harmonic oscillators. These potentials generate harmonic oscillations in all spacial directions defined by $\omega = \{\omega_x, \omega_y, \omega_z\}$, and a non-zero curvature in the potential can introduce changes to ω , making the atom oscillate at different frequencies. Thus, if each atom in a distribution of ultra cold atoms had different curvature’s in their local potential, a variable homogeneity arises.

Notice that in the observed dynamical phase diagram in figure 1.3, the collective magnetization was not observed for lattice sites with an inhomogeneity $\tilde{h}/2\pi < 2$ Hz. The proposed new quantum simulation would like to explore this region of near homogeneous fields in order to test for new quantum transitions between dynamical phases. To do so, the electromagnets used to produce magnetic fields for cold atom experiments must be able to produce strong magnetic fields while still having a

minimal curvature. It is also worth noting that, in an effort to make the magnets we design applicable to many sorts of quantum simulations, we want to engineer an electromagnet that is still able to control its gradient. This is because having a zero and non-zero gradient in the magnetic field can create new interacting dynamics for other quantum simulations that use different Hamiltonians. Though, our main concern is the magnetic curvature. Smale et al's experiment created a magnetic field with a curvature equal to 200 mG/cm^2 .

Thus, our goal is to create a magnet with a curvature that is less than 200 mG/m^2 , in order to further the efforts made in Smale et al's paper. Then, we also want to explore the relation between field strength and field curvature an electromagnet for cold atoms. While we have the goal of reducing the inhomogeneity by reducing the curvature of the magnetic field, we want to ensure that our magnet is still capable of producing strong magnetic fields capable of simulating interaction strengths J near and at the Feshbach resonances.

We then set a goal of designing a magnet with 832 gauss of field strength with as little curvature as possible, where 832 gauss is estimated to be the Feshbach resonance for lithium-6 the atom we are concerned with in our quantum simulator [9].

2.2 Why? The search for a quantum critical point

Previous work may have lacked the ability to reach the near-zero inhomogeneity range of the dynamical phase diagram, but why does that matter? Well, it is hypothesized that at this point, lies a quantum critical point (QCP), that uses quantum fluctuations to governs the boundary between a paramagnetic and a ferromagnetic quantum phase in the figure 1.3. This is another transition between dynamical phases like described in the Smale et al [7], however at a much lower inhomogeneity and interaction strength.

In principle, if our magnet is able to reach near-zero inhomogeneity and interaction strength, the Heisenberg model reduces to zero. At which point, other weaker interactions can be observed as their behaviours won't be overshadowed by the two strong interactions in the Heisenberg model. In this limiting case, the Heisenberg quantum simulation ceases to be a valid model as new interactions, like dipole-dipole interactions, or dynamics arising from the gradient of the magnetic field, would be observable. From that point, a new Hamiltonian model could be engineered to un-

derstand a potentially new quantum phase of matter.

Hence, the potential of observing new quantum matter behaving under different Hamiltonian's. To this end, not all quantum critical points are not yet fully understood in contemporary physics. Though it is suspected that understanding mechanisms that occur at QCPs could lead to better understanding certain forms of superconductivity, which is certainly a strong motive for conducting lots of contemporary physics due to its potentially applications in electronics, and quantum information.

2.3 Nullifying the curvature

Acquiring a homogeneous magnetic field usually employs a design called Helmholtz coils. This design consists of two solenoids of radius r that are spaced a distance h apart. Helmholtz coils can be constructed such that when $h = r$, the magnetic fields at the center becomes uniform up to the second derivative, i.e. $\frac{\partial^2 B}{\partial x^2} = 0$. If the coils are spaced a larger distance apart, the magnetic field at the center weakens and thus gains a positive curvature. If the coils are spaced at a smaller distance apart, then we arrive at a negative curvature. However, recreating this uniformity, and especially at the degree required for this quantum simulation, is difficult for multiple reasons.

The Helmholtz coil homogeneity estimation is an analytical solution that assumes infinitely thin wires for the coils, as it is derived from the Biot-Savart law. This is not physically realizable, as real electromagnets have finite dimensions that complicate the $h = r$ condition. Then of course, the electromagnet used in cold atom experiments has a current that runs in a $\propto 1/r$ distribution over the cross sectional area of the wire, which again, is incompatible with the infinitely thin wires defined by the Biot-Savart law.

Even if we knew what radius of finite-thickness electromagnets would approximate Helmholtz coils, the cold atoms apparatus has size constrictions that complicate changing the radii or separation distance such that two solenoids cannot be spaced at a distance equivalent to their radius. A lower limit on radii is put in place due to the need for optical cooling and trapping of cold atoms at the center of the electromagnet ensemble, the electromagnets need to have optical paths from multiple directions that are wide enough to accompany the diameter of the optical beams.

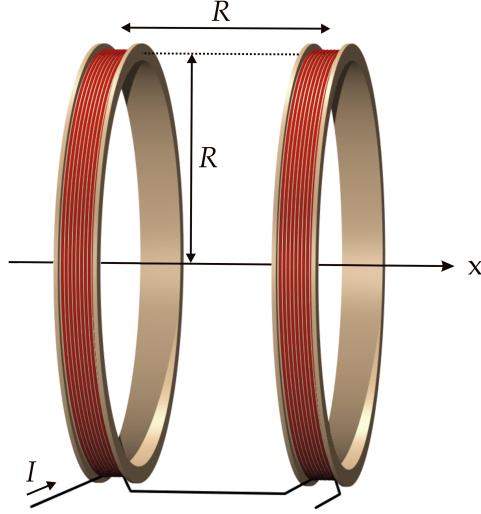


Figure 2.1: Two solenoids in a Helmholtz configuration, where the distance between them is equivalent to their radius. (reprinted from wikimedia page for helmholtz coils with creative commons license by SA-2.0 permission)

Then, there is an upper limit on radii of electromagnet because the apparatus is confined to a vacuum chamber. Expanding experiment to accommodate for this size constraint would be time-consuming and costly. Even if the vacuum chamber were larger, an electromagnet with increasingly large radii would then require larger currents to create sufficiently strong magnetic fields at the center of the apparatus. At which point, the larger currents may cause power dissipation issues as the coolant system fails to keep temperatures low. Thus, there remains a need to generate a homogeneous magnetic field, while restricting the electromagnet to a range of geometries.

In response to the problem of homogeneity in magnetic fields, we propose a electromagnet design that utilizes a secondary pair of Bitter-type electromagnets to nullify the curvature in the magnetic field. The secondary pair of Bitter-type electromagnets that will have a smaller radius than the bias coils but still fits within the restraints created by the optical bore. Additionally, the closer proximity allows us to run a weaker and opposite current that allows the magnet to produce little deviation to the field strength and gradient, but just enough magnetic curvature to cancel the curvature of the main "bias" coils' fields. Henceforth, I will call the main outer electromagnets as the "bias-coils" and the additional inside coils as the "curvature-coils".

$$\begin{aligned}
B_{tot} &= B_b + B_c \\
\frac{\partial B_{tot}}{\partial r} &= \frac{\partial B_b}{\partial r} + \frac{\partial B_c}{\partial r} \\
\frac{\partial^2 B_{tot}}{\partial r^2} &= \frac{\partial^2 B_b}{\partial r^2} + \frac{\partial^2 B_c}{\partial r^2} = 0
\end{aligned}$$

The working principle is that the total magnetic field, B_{tot} , can be separated as the field of the bias coils B_b and the field of the curvature coils B_c . In order to be deemed as homogeneous, we require that the second derivative of the field, or magnetic curvature, is zero. Thus in order to arrive at zero curvature in the total magnetic field, we can create a bias and curvature coils that cancel out the curvature of the magnetic field. In addition to that goal, we want to create a magnet that has variable configurations that allow for more diverse magnetic profiles that can be used for other quantum simulations—like for example, being able to tune the gradient of the magnetic field.

More over, we aim to produce a relation between the currents of two magnets. Given some geometry, we want to construct an operational profile that defines the currents of the two magnets in order to produce a particular field strength and field curvature. Consider Biot-savart's law:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_C \frac{\mathbf{I} d\mathbf{l} \times \hat{\mathbf{r}}'}{|\mathbf{r}'|^2}$$

Notice that the magnetic field is proportional to the magnitude of the current running through an electromagnet. This ensures that both the magnetic field strength and the magnetic field curvature are proportional to the current. Through this capstone, we aim to quantify this proportionality, potentially allowing us to create the homogeneous and strong magnetic fields we desire.

2.4 Bias-Curvature coils geometry

2.4.1 Types of electromagnet design

One commonly used electromagnet in cold atom experiments are called Bitter-type electromagnets. Cold atoms benefit from this Bitter-type electromagnets because

they have a much higher threshold of maximum field strength. In fact the strongest manmade magnetic field, is of a Bitter-type design.

Traditional electromagnets are resistive, meaning current is run through wire with some resistance. For example, a solenoid winds conductive material helically so that a magnetic field is produced in its axial direction. Its strength, calculated from Ampere's law, says that $B = \mu_0 NI/L$, where B is the magnetic field, μ_0 is the magnetic permeability constant, N is the number of turns on the solenoid, I is the current running through the wire, and L is the length of the solenoid. While the magnetic field is, ideally, linear with the current running through it. The current cannot be realistically increased without bound in resistive magnets. When a high current is run through a wire, Joules first law states that the power of heating in a wire, $P = I^2 R$. Additionally, the resistance of conducting material tends to increase with higher currents. The change in temperature is proportional to P , thus the heating of a solenoid is of order $> O(I^2)$. For many applications, the high temperature release of high-current resistive magnets presents a risk of material failure.

A common work around this issue is by developing Bitter-type electromagnets, which abstracts the helical design of a solenoid into layers of current carrying disks, separated by insulating material. Then, to combat the heating issues present in solenoids, the stacked quality of these disks allow coolant to be pumped axially through the solenoid. Then, this resistive magnet is capable of producing much stronger magnetic fields, granted that the magnet is appropriately cooled.

2.4.2 Modified Bitter-design

In a paper by Sabulsky et al, the researchers propose a modified design of Bitter-type electromagnets that is specialized for cold atom experiments. It produces a high magnetic field output and retains efficient heat removal properties like most Bitter-type electromagnets. The novel design feature discussed in the paper is how the coolant is pumped through and pumped out on the same side of the electromagnet, which is important given that one side of the electromagnet is reserved for the cold atoms. Instead of water being pumped axially like conventional Bitter-type electromagnets, the water is pumped and enters ducts between the copper layers that allow the water to travel along the polar angle, while maintaining surface contact to two copper layers. Additionally, the design introduces gaskets to seal these ducts

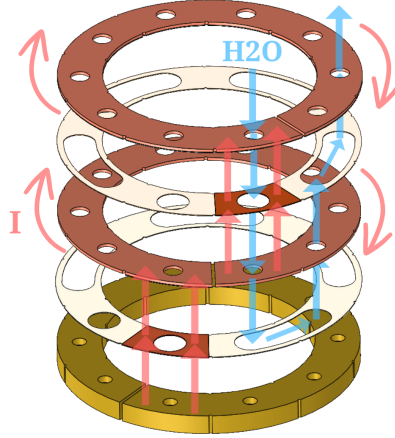


Figure 2.2: The Bitter-type electromagnet design from Sabulsky et al [10] but reconfigured for our slightly larger magnet, which needed more water-cooling ducts to maintain a good filling fraction λ . Yellow layer is made of brass, red layer is copper, and light beige is the polyester spacers. This figure does not include, the gaskets that seal the water ducts in the spacer layer, or the titanium rods that align and compress the magnet.

in order to ensure water-tight plumbing without epoxy. When this design had 400 Amps running through it, the paper reported that the temperature only increased by 7°C under continuous water-cooling with a water flow of 3.8 liters/min.

2.4.3 The initial design

For our purposes of reducing the inhomogeneity, we require four electromagnet ensembles that we will call: the top bias, the bottom bias, the top curvature, and the bottom curvature coils. Each of which utilize the modified Bitter-type configuration. In figure 2.3, we detail main design features of the modified Bitter-type coils.

Incomplete design

The radii and widths of the bias and curvature coils is yet to be determined because it is unsure what combination of geometry and currents running through the coils can produce the most strong yet homogeneous magnetic fields. And as mentioned early, it is unclear how the geometric Helmholtz condition manifests with the modified Bitter-type design, given its thickness. Though, if we assume that the radius is the mean radius of the magnets two radii, and that the height is measured starting from the half of its height to the center of the apparatus, we can get an estimate

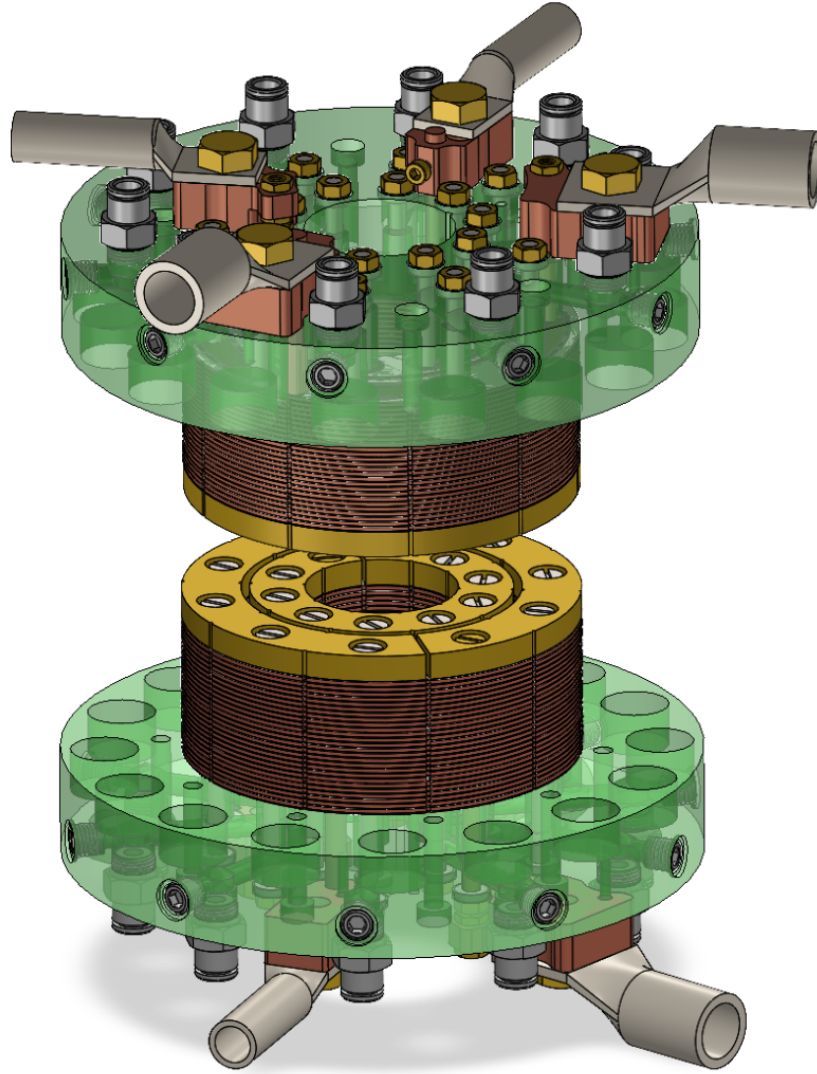


Figure 2.3: Pictured is a prototypical design of the Bias-Curvature coils. In addition to the four Bitter electromagnets in the center, we display a green fiber glass cap which mounts the magnets. This part is necessary for tightening the magnets, having a means of electrically connecting them, and water-cooling the magnet.

of how Helmholtz condition can be met. The coils are fixed at being ≈ 29 mm apart (measured from their brass caps), while the coils can be compressed to a maximum height of ≈ 39.1 mm. This means that the mean radius of a magnet has to be ≈ 68 mm to be Helmholtz by our assumptions. However, this design is not possible—

We seek to design these magnets so that it fits in the context of an ultra cold atoms experiment, where atoms under the influence of our magnet's field must be in a vacuum. As seen in figure 2.4, a vacuum chamber surrounds and constrains our magnet. Given the ultra cold atom apparatus, we fix the spacing between the top and bottom coils as 29 mm. The diameter of the holes for water-cooling is fixed to 6 mm which is just large enough to host screws for alignment of the coils and provide room for water to be pumped through. Lastly the most inner radius and most outer radii of the coils are confined range between 14 mm and 48.5 mm. The vacuum chamber imposes these restrictions in order to leave optical paths for various lasers for the trapping and cooling of atoms. Given the size of the vacuum chamber, our magnets cannot exceed 48.5 mm in radius, failing to meet the Helmholtz criteria hypothesis. It is then also too costly and time consuming to modify the extended experiment to accommodate a larger magnet if we wanted to meet this Helmholtz criteria. Lastly, a larger magnet also produces weaker fields given some current, since magnetic fields drop in strength $\propto 1/r$.

Instead of meeting the unfeasible Helmholtz condition, we must consider other radii to try and generate Helmholtz-like conditions. We start by likely making the bias coils as large as possible to approach the Helmholtz condition. Then we ask, at what radius should the inner curvature coils be put, and what current should they run relative to the bias current, in order to make a sufficient correction to the bias coils' inhomogeneity.

We will attempt to answer this question through simulation in the next chapter.

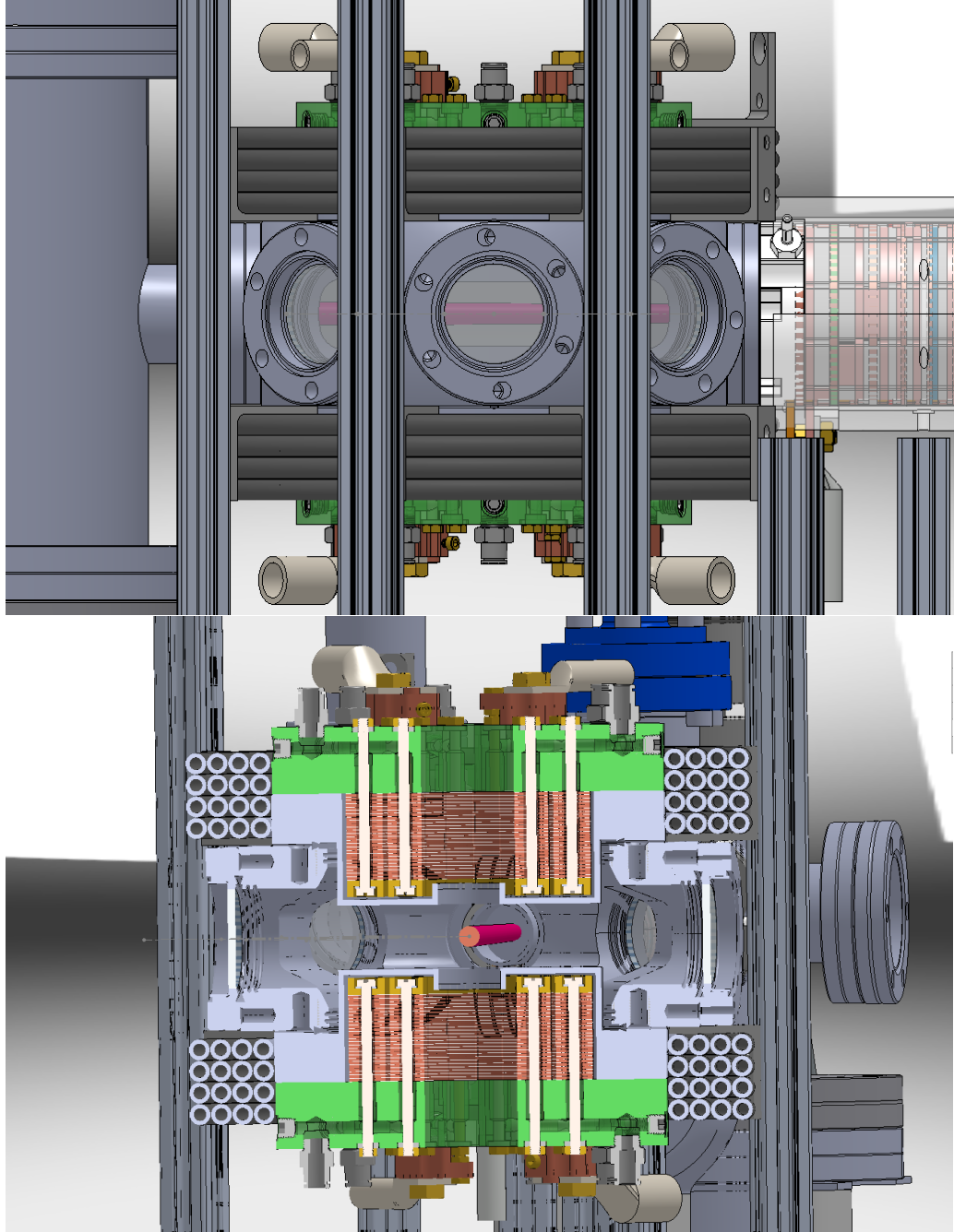


Figure 2.4: Pictured is a 3D CAD model of vacuum chamber that constrains the electromagnet. The top figure shows the vacuum chamber windows that feature the optical paths that the beams travel, while the bias-curvature cap protrudes from the top and bottom. The surrounding layers construct another magnet called the MOT coils, another necessary magnet for this quantum simulation. The spliced view at the bottom better shows how the magnets are isolated from the vacuum chamber but constrained by it.

CHAPTER 3

Simulation

Through simulation, we aim to iterate over different geometries of the bias-curvature coils to give us the best control over magnetic field output, particularly for magnetic field strength and inhomogeneity. In order to do this, simulations must be fast and efficient, to allow many simulation iterations using differently sized electromagnets.

3.1 Weak formulation for magnetostatics

In order to computationally compute the magnetic field, we employ the weak formulation of magnetostatics. The weak formulation is a mathematical tool from the calculus of variation that will help us calculate $\vec{\mathbf{B}}$. Before doing so, we also introduce the magnetostatic potential $\vec{\mathbf{A}}$, a scalar field that is defined by $\vec{\mathbf{E}} = \nabla \times \vec{\mathbf{A}}$. We can apply tools from the weak formulation to magnetostatics, and it allows us to solve for $\vec{\mathbf{A}}$ by turning the problem into a optimization problem for some variable, rather than a complicated boundary value PDE problem. Then after arriving at $\vec{\mathbf{A}}$, we can take its curl to find the magnetic field $\vec{\mathbf{B}}$

To arrive at the weak formulation, we start at the description of electromagnetic phenomena—Maxwell’s equations. These equation model the electromagnetic field by relating the electric field $\vec{\mathbf{E}}$, magnetic field $\vec{\mathbf{H}}$, the electric displacement field $\vec{\mathbf{D}}$, and the magnetic flux density $\vec{\mathbf{B}}$.

$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial \mathbf{B}}{\partial t} \quad (3.1.1)$$

$$\nabla \cdot \vec{\mathbf{D}} = q \quad (3.1.2)$$

$$\nabla \times \vec{\mathbf{H}} = \vec{\mathbf{J}} + \frac{\partial \mathbf{D}}{\partial t} \quad (3.1.3)$$

$$\nabla \cdot \vec{\mathbf{B}} = 0 \quad (3.1.4)$$

In order to solve these differential equations for our bias-curvature magnet, assume that $\vec{\mathbf{B}}$ and $\vec{\mathbf{D}}$ is time-invariant. Thus, $\frac{\partial \mathbf{B}}{\partial t}$ and $\frac{\partial \mathbf{D}}{\partial t}$ simplify to zero. Then, we can focus on the differential equations 3.1.3 and 3.1.4 to solve our magnetic statics simulations.

We also note that how the magnetic field and its dual field can be expressed as $\vec{\mathbf{B}} = \mu \vec{\mathbf{H}}$ where μ is the permeability of a material. Though, this simulation does not concern itself with distinguishing $\vec{\mathbf{B}}$ and $\vec{\mathbf{H}}$, seeing as the only magnetic field that is relevant is that at the center of the apparatus, which is in a vacuum.

Those equations can be put together to form what is known as the primal canonical equation, which define how the magnetic field behaves on some domain Ω [11]

$$\left. \begin{aligned} \vec{\mathbf{B}} &= \nabla \times \vec{\mathbf{A}} \\ \vec{\mathbf{B}} &= \mu \vec{\mathbf{H}} \\ \nabla \times \vec{\mathbf{H}} &= \vec{\mathbf{J}} \end{aligned} \right\} \text{ on } \Omega$$

Then, several mathematical tools in variational techniques can be deployed to construct a type of function called functional $\Pi^A[\vec{\mathbf{A}}]$. This functional is constructed such that its stationary point is the solution to the primal canonical equation, i.e. $\partial \Pi^A[\vec{\mathbf{A}}; \vec{\mathbf{A}}_*] = 0$. Here, $\vec{\mathbf{A}}_*$ denotes a small variation in the magnetic field function. This paper will not concern itself with that full derivation but can be found in a variational techniques in electromagnetic textbook by J. Penman [11], or in a paper on variational formulations of magnetostatics by [12]. In summary, the primal canonical equations combine to form a functional known as the complementary energy variational formulation.

$$\Pi^A[\vec{\mathbf{A}}] = \int_{\Omega} \frac{1}{2\mu} (\nabla \times \vec{\mathbf{A}}) \cdot (\nabla \times \vec{\mathbf{A}}) d\Omega - \int_{\Omega} (J \cdot \vec{\mathbf{A}}) d\Omega$$

Then the solution to the stationary point of this functional gives us the weak formulation of 3D magnetostatics.

$$\int_{\Omega} \frac{1}{\mu} (\nabla \times \vec{\mathbf{A}} \cdot \nabla \times \vec{\mathbf{A}}_* + \nabla \cdot \vec{\mathbf{A}} \nabla \cdot \vec{\mathbf{A}}_*) - \int_{\Omega} J \cdot \vec{\mathbf{A}}_* = 0 \quad (3.1.5)$$

Note that the $\nabla \cdot \vec{\mathbf{A}} \nabla \cdot \vec{\mathbf{A}}_*$ term does not arise from the weak formulation but another term that is added to deal with gauge invariances in the magnetic potential. Because the magnetic potential is non-unique, the extra term is a commonly include term that forces the weak formulation to solve for a particular solution Creusé et al [13].

3.2 FreeFEM++

Now that we have the weak-formulation of the magnetostatic problem, the problem becomes easier for computational techniques to approximate the solution [14]. FreeFEM++ is a free finite element methods module that is programmed to quickly and efficiently solve weak formulations like ours in equation 3.1.5.

FreeFEM employs the finite element method, which discretizes a function Φ which is defined on some domain Ω , into individual elements, and weighs them in a sum according to

$$\Phi \approx \tilde{\Phi} = \sum_i \Phi_i \cdot N_i \quad (3.2.1)$$

Where Φ_i is the solution to the weak formulation at a particular partion of Ω , and then N_i is a shape function that serves as a weighting coefficient in the sum [15]. The magnetic potential $\vec{\mathbf{A}}$ serves as our discretized and approximated function. Solving the weak formulation at each of the partitions of Ω then allows FreeFEM++ to do some linear algebra tricks to arrive at the final estimation for $\vec{\mathbf{A}}$.

Below is our snippet of our implementation of the weak formulation in FreeFEM++ from equation 3.1.5. Adapted from the FreeFEM Magnetostatics demo module.

```
//Problem
varf vLaplacian ([Ax, Ay, Az], [AAx, AAy, AAz])
  = int3d(Th)(
    Nu * Curl(Ax, Ay, Az)' * Curl(AAx, AAy, AAz)
    + (1./Mu) * Divergence(Ax, Ay, Az) * Divergence(AAx, AAy, AAz)
  )
- int3d(Th)(
  [Jx, Jy, Jz]' * [AAx, AAy, AAz]
)
+ on(BoxWall, Ax=0, Ay=0, Az=0)
;

matrix<real> Laplacian = vLaplacian(Ah, Ah, solver=sparsesolver);
real[int] LaplacianBoundary = vLaplacian(0, Ah);
Ax[] = Laplacian^-1 * LaplacianBoundary;

//Magnetic induction
Ah [Bx, By, Bz];
[Bx, By, Bz] = Curl(Ax, Ay, Az);

//Magnetic field
Ah [Hx, Hy, Hz];
[Hx, Hy, Hz] = Nu * [Bx, By, Bz];
```

3.3 Meshing

In our magnetostatics problem, the domain Ω is a sphere with a radius large enough to encompass the entire bias-curvature magnet. The simulation in the finite element method then requires a 3D-mesh defined on Ω . The finite element simulation rely on 3D meshes to partition a complicated geometry into discrete tetrahedra; our simulation computes the magnetic field output generated by some n th tetrahedra that is embedded in our bias-curvature coils. As detailed in the FreeFEM++ section,

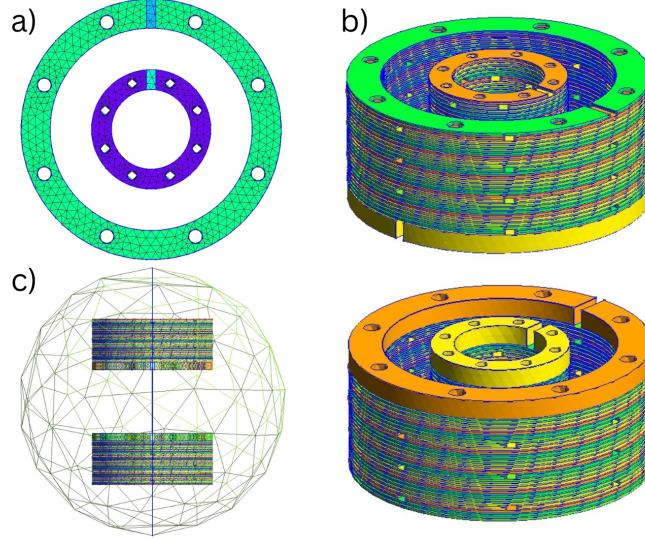


Figure 3.1: a) a figure representing how the meshing algorithm partitions a surface into triangles. b) a figure representing how the meshing algorithm partitions a volume into tetrahedra. c) A figure showing how the domain Ω of the simulation extends in a sphere around the magnet, but strictly speaking, only the magnetic field at the center matters. This is why there is a vertical line at the center of the sphere—there is a lot of tiny points here so that tetrahedra generated near the center axis of the mesh are small, to ensure precise computational estimates.

these tetrahedra make up the nodes that Φ_i s and N_i s are calculated at in the linear approximation of $\vec{A}$

For this step, I created parametrically defined meshes using GMSH, a free finite element mesh generator. The code for such can be found in the appendix B [5.4]. The parametric definition is a choice to allow for iterable simulations, where each simulation runs with different geometries to define the bias-curvature coils.

An important description of your mesh is the mesh size, which effectively determines the number of partitions of Ω there are, and subsequently the accuracy of the simulation. While minimal mesh sizes would make our simulation less of an approximation, we ought to limit the mesh size in order to prevent intense computational power and run-time. Often, the marginal approximation is negligible beyond some point. We took an adaptive approach to meshing so that the mesh gets finer in regions where our magnet simulation most matter. For example, we embedded a very fine mesh size that went along the z-axis at the center of the magnet. This was constructed

so that the magnetic field simulations would have high spacial resolution along the z-axis of the magnetic field, and then we can plot the subsequent field profile along that direction.

3.4 Levels of approximation

3.4.1 Choice of finite element

There are many types of finite elements, that are used to approximate the shape functions and weightages in equation 3.2.1. We employed P0 in our simulation, a piece wise constant and discontinuous finite element. There are more accurate finite elements, like the piece wise linear, or quadratic finite elements. Though, preliminary testing with these finite elements yielded differences in the simulation output that were negligible in the context of our problem. More importantly, the more rigorously defined finite element lengthens computational time by a lot.

3.4.2 Simplifying the geometry for currents

The bias-curvature coils have very complicated geometries. While it was possible to recreate the entire geometry in the mesh, the simulations often took too long to compute—on the order of hours. The code could be sped up using parallel processing or implementing more advanced memory usage algorithms in FreeFEM, however our limited expertise prevent that.

We also had issues when defining the current for the full geometries as the thin layers of copper-coils could not be easily defined given the FreeFEM package. The mesh defined the bias-curvature as a single volume V , inside of which a current was defined along the polar angle. However, when the layers are so thin, the mesh generation does not create nodes strictly inside the borders of V unless the mesh size was very small. Thus, if the mesh size is too large, there would be no nodes defined inside the thin layers of the magnet to define a current, as seen in figure 3.3. Simply, put, the mesh size could not be reduced to meet this requirement given our computational limitations.

Instead we opted to simplify the geometry of the magnet for the purposes of simulation. We did away with the multiple layers of the Bitter-type magnet and the keystones that carry the current upwards. Instead, we treated each of the four elec-

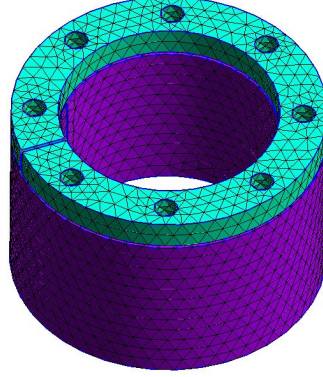


Figure 3.2: The simplified geometry of one Bitter electromagnet ensemble. The layers of the electromagnet were removed in favor of one continuous electromagnet. The holes in the copper for water cooling remained, and the copper (in purple) was still kept separate from the brass (in blue).

tromagnets as one large copper cylinder that had current running along the polar angle. Though the design was significantly simplified, we were still able to maintain the holes of the electromagnets in these simulations.

Elaborating further on how current was defined, we approximate the current density vector $\vec{J}$. Normally current would be better modeled by an average current density $\vec{j}(r) = I/(d\lambda\ln(R_{out}/R_{in})r)$, where d is the thickness of a layer in the magnet, λ is the filling fraction of the layer, I is the current, and r is the radius. To ease computational power, we assumed a constant current density $J = I/d(R_{out} - R_{in})$. Since the volume of the copper increased with the simplified geometry, J should be adjusted, but we assumed, given the small thickness of these layers, that the contribution would be negligible. With the simplified the geometry, current now ran purely in the polar direction, neglecting off asymmetric field contributions by the copper keystone layers.

3.5 Computational methodology

We now explain how we use the techniques discussed above were used to answer our research question: What geometry of bias-curvature magnet best suits our control of the magnetic field, namely its field strength and field curvature.

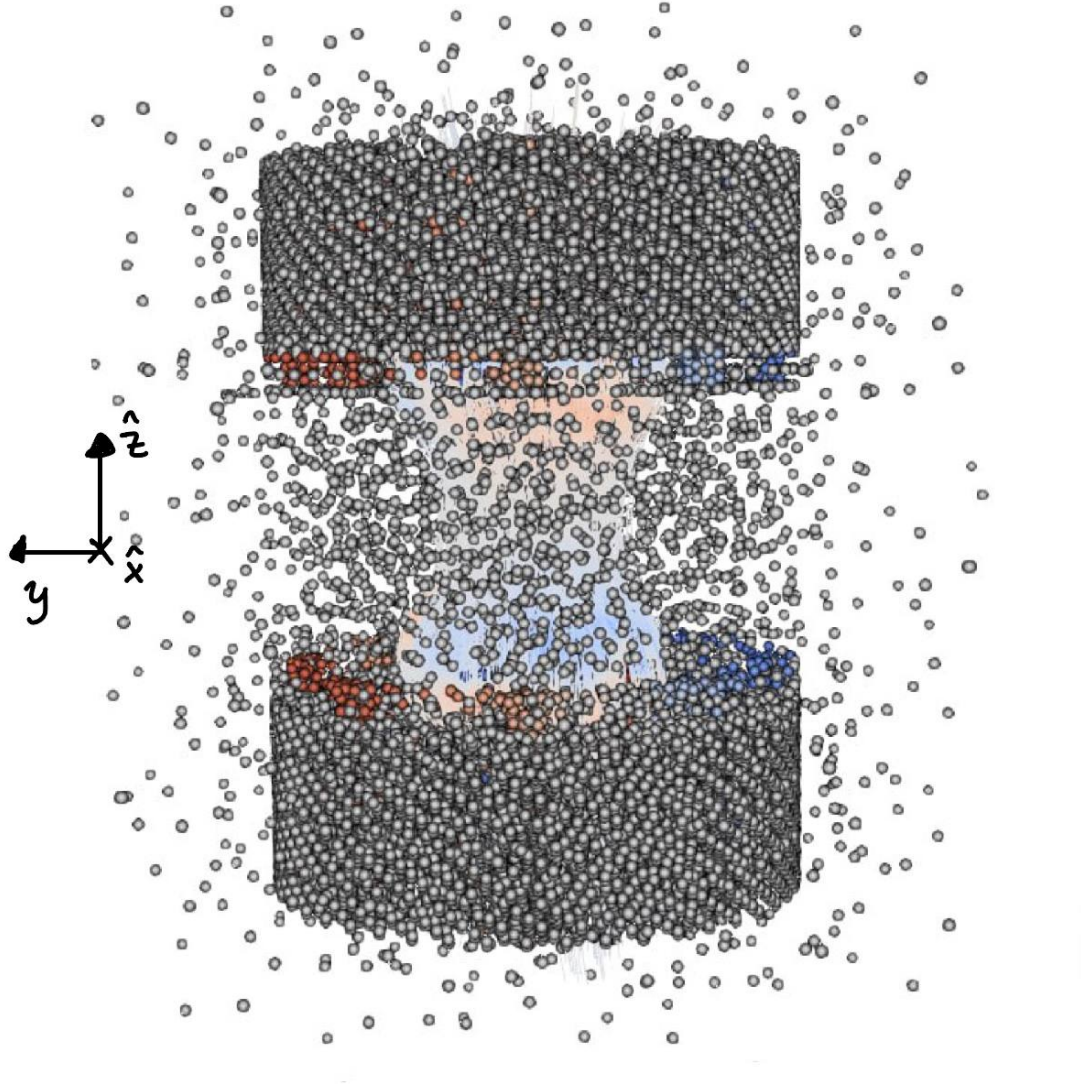


Figure 3.3: A screenshot from Paraview, a software used to visualize the magnetic field lines generated by the simulation. The circles represent node points in the simulation and their color represents their current density in the x direction. Blue represents a negative x component of current while red is positive, thus the current is running clockwise if x is running into the page. Additionally, the streamlines in the center of the simulation represent magnetic field lines colored by the same convention for the z component of the magnetic field. Notice how the copper layers failed to have any defined current, while the brass layers did. Then the simulated magnetic field profile consisted only of the brass layer's contributions.

3.5.1 Variable parameters

Through simulation, we can vary the parameters of the mesh that define the geometry of the coils and computationally analyze how they affect the magnetic field profile. Some geometries of the magnet are fixed, like how the height of our magnet is immutable because of the number of layers in our magnet and material thicknesses available. Then we also mentioned how the vacuum chamber limits the radii of the magnet.

Given those restrictions, the main parameters we optimize in this capstone are the various radii that define the electromagnet ensembles, and the current that is run through them. We aim to create a simulation that considers some geometry in the form of the radii of bias-curvature coils and numerically computes its magnetic profile and thus its inhomogeneity. The current in the simulations does not have any restriction as a variable parameter. This is because the current should be linear with the magnetic profile, thus if we produce the magnetic field profile of a magnet with a particular geometry, we can find the magnetic profile of that magnet running at any current by multiplying the magnetic profile by some factor. Consider biot-savart's law for a finite thickness conductor, which is another valid formalism for calculating the magnetic field but is simply computationally expensive to implement. This formalism shows that the current can be factored out of the magnetic field definition, and it is multiplied by some function $G(z)$ that depends purely on the geometry of the magnet. Again, we can only do this because I is trivially assumed to be in the polar direction $\hat{\phi}$ and any magnetic field contribution in the $\hat{x}$ and $\hat{y}$ directions are canceled out. Though, we should also bear in mind that the current physically cannot be too large, depending on the flow rate of cooling, as high currents can lead to large changes in temperature that are dangerous for the electromagnet.

$$\begin{aligned}
\vec{B}(\vec{r}) &= \frac{\mu_0}{4\pi} \iiint_V \frac{(\vec{J}dV) \times \vec{r}}{|\vec{r}|^3} \\
&\approx \frac{\mu_0}{4\pi} \sum_{V_i=1}^N \frac{(\vec{J}\Delta V_i) \times \vec{r}}{|\vec{r}|^3} \\
&\approx \frac{\mu_0}{4\pi} I \sum_{V_i=1}^N \frac{(\Delta V_i \hat{\phi}) \times \vec{r}}{d(R_{out} - R_{in})|\vec{r}|^3} \\
&\approx \frac{\mu_0}{4\pi} IG(z)
\end{aligned}$$

We can then consider a simulation S that inputs the geometry of the bias-curvature coils and maps to an inhomogeneity $\tilde{h}$ that we desire to minimize. Note that for our simulations, it is also sufficient to minimize the magnetic field curvature, since calculating $\tilde{h}$ would require the extended optical set up to support the optical harmonic trap.

$$S : \{R_{in}^B, R_{out}^B, R_{in}^C, R_{out}^C\} \mapsto \tilde{h} \quad (3.5.1)$$

Here R_{in}^B and R_{out}^B denote the inner and outer radii that define the top and bottom bias coils and R_{in}^C and R_{out}^C denote the radii for the the top and bottom curvature coils. We mentioned earlier the constraints on our design in section 2.4.3. This is expressed by an ordered relation that combinations of radii that our magnet can have:

$$14\text{mm} < 28\text{mm} < R_{in}^C < R_{out}^C < R_{in}^B < R_{out}^B < 48.5\text{mm} < 97\text{mm} \quad (3.5.2)$$

According to simulation 2.4.3, the simulations' variable parameters are constrained by 14mm and 48.5 mm. However, we mentioned 28mm and 97mm here as constraints as well. This fact is purely an accident and resulted because of confusion between the the radius vs diameter of the magnet. The simulations presented from this point on restrict the radii to 28mm and 97mm when in fact, it is the diameter of the magnets that are restricted to those dimensions. Unfortunately, the results of these simulations were produced before my computer died, effectively loosing all of my data. Regrettably, we only realized this mistake too late in the production of this capstone, and we didn't have enough time to recreate these simulations given their long run time. Ultimately this mistake made it impossible for bias coils to be close to helmholtz conditions, and instead, allowed the curvature coils to be near helmholtz conditions. This is likely why later, it seems that the curvature coils has less curvature than the bias coils in the optimal bias-curvature simulation result, as shown in figure 3.5.

3.5.2 Using linearity of fields

The simulations employ another trick in linearity to simplify and speed-up our simulation. Given the ordered relation, the combinations of radii for our four defining radii are too many to simulate. To simulate the four electromagnet ensembles with radii combinations that span as many permutations would require running S with some redundancies in computation. For example, if S_1 is run once to compute the magnetic profile of the bottom bias magnet which has radii R_1 and R_2 , another simulation S_2 may have to calculate the magnetic profile of the top curvature magnet with the same radii R_1 and R_2 . These two magnetic profiles are identical, however, are computed twice because they were used for different magnets in different simulations.

It is simpler and quicker to instead perform S under all permutations of R_{in}, R_{out} for a single electromagnet ensemble. Then, linearly add those pre-generated magnetic profiles to create all combinations of radii that span the possible state space. This includes flipping a magnetic profile across the center of the apparatus to mimic a top and bottom electromagnet. By looking at the magnetic profile of all these combinations of radii, we can then observe what radii tend to produce minimal magnetic curvature and thus inhomogeneity.

As mentioned before, the magnetic profiles are linear with I . Then, given each combination of electromagnets with some set of radii, we can find the best current configuration that would manifest a minimal curvature by multiplying the magnetic profile over many different currents and selecting the ideal one.

3.5.3 Curvature estimation—Savitzky-Golay Filter

As outlined in the design chapter, we also concern ourselves with the gradient and curvature of the magnetic field, since those tunable parameters allow us to probe different behaviours for quantum simulations. We can estimate the derivatives of the magnet by using a Savitzky-Golay filter

The Savitzky-Golay filter is an algorithm to smooth discrete data. It can be even used to calculate the derivatives for noisy data sets. When working with data, and especially in simulation, a noise floor in the data makes using a $f'(x) \approx \frac{f(x+dx)-f(x)}{dx}$ approximation difficult. In the presence of noise, the data can be over or underestimated when the step size is on the small enough to

view noise fluctuations. The ideal workaround is to smooth the data first, and then take the derivative. The Savitzky-Golay filter provides an easy way of applying this estimation. We used the savitzky-Golay convolution coefficients for a cubic polynomial of window size 5 and step size h , which should be sufficient to estimate the first and second derivative locally at the center of our apparatus.

$$f'(x) \approx \frac{1}{12h} \left(f(x-2h) - 8f(x-h) + 8f(x+h) - f(x+2h) \right)$$

$$f''(x) \approx \frac{1}{7h^2} \left(2f(x-2h) - f(x-h) - 2f(x) - f(x+h) + 2f(x+2h) \right)$$

3.6 Simulation results

We conducted simulations S to generate the magnetic profile, and curvature, for 80 different geometries of magnet. 40 of which were dedicated to testing the effect of mean radius, 40 were dedicated to testing the effect of magnet width. Note that, in general, the magnetic profiles produced by simulation are likely not an accurate estimation, by orders of magnitude. In preliminary investigations, it seemed that the simulations, which should be estimating B in tesla, underestimates the expected magnetic field for a given current. Though, the profile shape is comparable to experiment results later, which justifies a comparative analysis of magnetic profiles to investigate relationships rather than actual field strength measurements.

3.6.1 Bias-Curvature mean radius simulation

40 of the simulations used 40 pairs of (R_{out}, R_{in}) to define an electromagnet to mesh and integrate over. The pairs were defined such that the mean radius of the magnet increased by ≈ 13 mm with each simulation, while maintaining a constant width $R_{out} - R_{in} = 16$ mm. Figure 3.4 plots all 40 of the simulated magnetic field profiles, assuming a current of 500A. The figure demonstrated how the field profile made a narrower peak with smaller mean radii, and the field broadened with larger mean radii. Then, the x,y components of the magnetic field are as expected, negligible.

Written in millimeters, the following radii were considered:

$$R_{out} = \{45, 46.3, 47.7, \dots, 97\}$$

$$R_{in} = \{29, 30.3, 31.7, \dots, 81\}$$

By using this pre-generated data set of magnetic profiles, we could construct the total bias-curvature magnet with combinatorics.

$$\begin{aligned} B_{BC}(z) = & B_B(z)I_B + B_C(z)I_C \\ & + B_B(-z + c)I_B + B_C(-z + c)I_C + \end{aligned}$$

Where subscripts B_B denotes the magnetic profile of the bias coil, B_C of the curvature coil, and B_{BC} of the total bias curvature coil. I_B and I_C are the currents for the bias and curvature coils. Note that the B_B are generated given some geometry (R_{out}^C, R_{in}^C) while B_C is generated by some geometry (R_{out}^B, R_{in}^B) such that the ordered relation in equation 3.5.2, in order to prevent overlap. the $-z + c$ parameter reflects the magnetic profiles across the $z=0$ in order to simulate the top and bottom curvature coils.

After permuting through all possible values for $R_{in}^B, R_{out}^B, R_{in}^C, R_{out}^C$, we tested B_{BC} for different values of I_B and I_C across the range of $(-500, 500)$ amperes. Specifically, we selected the ratio $I^* := I_C/I_B$ that minimized the curvature calculated with the Savitzky-Golay filter at $z = 0$.

The optimal design that this combinatorics simulation yielded produced the magnetic profile in figure 3.5.

A curious feature of the magnetic profile in figure 3.5 is that the curvature coil is actually running parallel with the bias coils, instead of running in the opposite direction, given that the current ratio is positive. It is also interesting that the curvature coils did not take the largest radius available. We initially expected that it would assuming that the larger radius would aid the magnet meet the helmholtz condition for homogeneity. For our curiosity, we wondered what the magnetic profile for a magnet with the largest available radii in the state space was, which is shown in figure 3.6. With this geometry, the optimal I_* was -0.247, implying that the coils

Magnetic field profile for different magnet mean radii

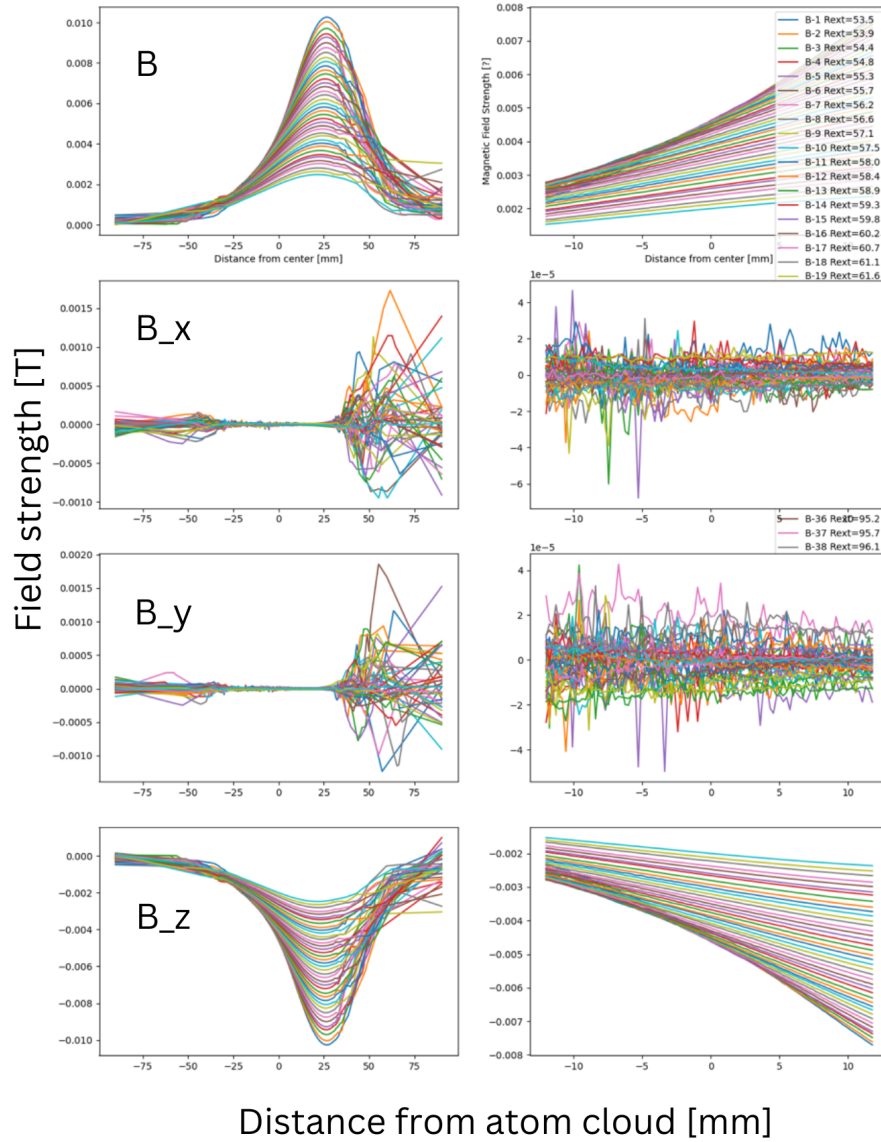


Figure 3.4: The magnetic profile of coils with 40 different radii. First row is the magnetic field strength in the z direction, and the following rows are the x,y,z contributions to the field. Left column views the full range of simulated data, and the right column selects a smaller x-range view near the atom cloud. Note that beyond $|x| > 25$ mm, the simulations get less smooth, as the mesh was more coarse past this point. This is fine given that the field at the center is the main concern. The legend essentially would describe how magnets with larger mean radii produce broader and weaker peaks.

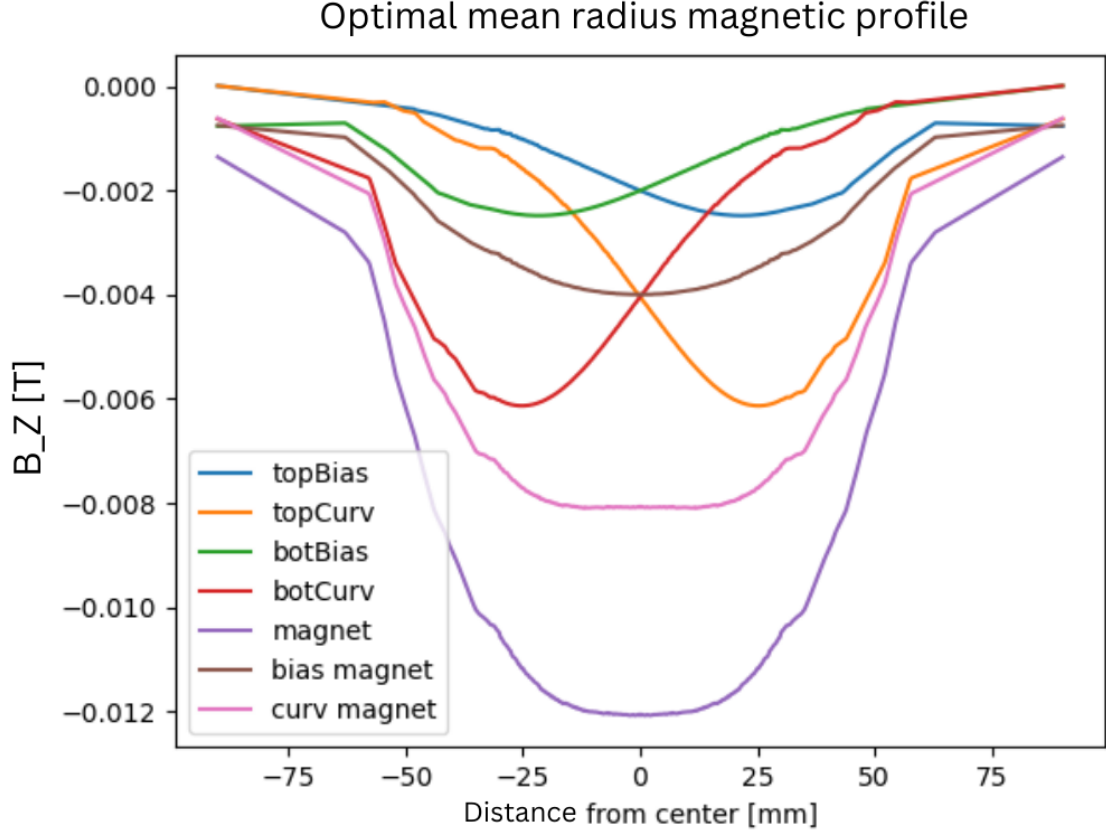


Figure 3.5: Magnetic profile of optimal mean radii, assuming a width of 16mm. The radii in question are a bias magnet with radii $R^B = (81 \text{ mm}, 97 \text{ mm})$ and a curvature coil with radii $R^C = (53 \text{ mm}, 69 \text{ mm})$. The current ratio $I^* = 0.990$. In this geometry, the curvature coil neutralizes the bias' curvature by 2 orders of magnitude. Though because of the mistake in parameter space for radii, it is more accurate to say that the bias coils are correcting the curvature of the curvature coils, seeing that the curvature coils radii include the helmholtz radius of 68 mm.

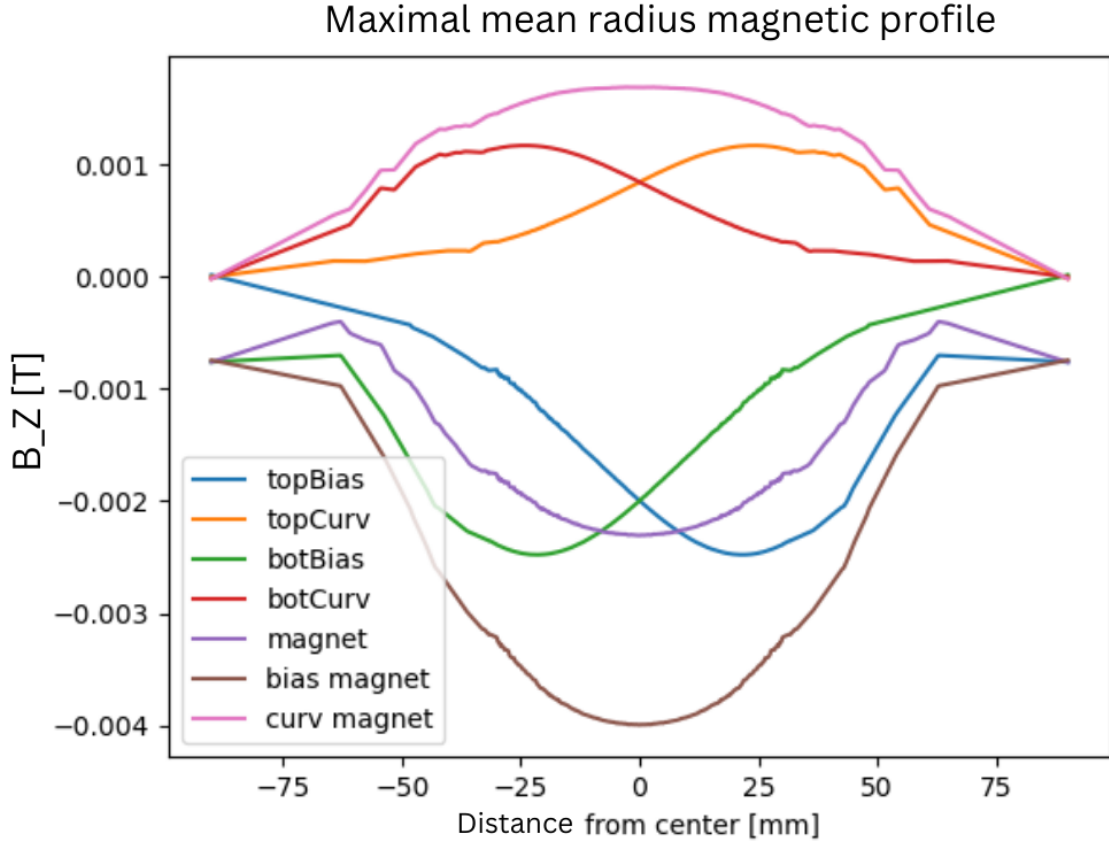


Figure 3.6: Magnetic profile of optimal mean radii, assuming a width of 16mm. The radii in question are a bias magnet with radii $R^B = (81 \text{ mm}, 97 \text{ mm})$ and a curvature coil with radii $R^C = (63.7 \text{ [mm]}, 79.7 \text{ [mm]})$. The current ratio $I^* = -0.247$. This creates a profile with a small positive curvature at 0. In this geometry, the curvature coil neutralizes the bias' curvature by 2 orders of magnitude.

here should work anti-parallel to approach a zero magnetic curvature! Though, note that while the curvature was minimized, this came at the cost of reducing the field strength of the total magnet, since the bias and curvature coils are acting against each other in that respect.

Unfortunately, the misinterpretation of the design constraints limit our simulation analysis from deducing the best magnet design for the experimental set up here. However we can draw some general conclusions for Bitter-type electromagnets of this design for larger radii that are embedded in larger vacuum chambers.

We note that our electromagnets can produce two types of magnetic profiles: The

kind where the top and bottom electromagnets produce two close and broad peaks that sum to a single peak at the center. The second kind is when the two peaks are far and narrow enough such that their sum still creates a two-peak magnetic profile. This distinction is important with the characterization of how these Bitter-type electromagnets interact. In figure 3.5, the magnetic profile is able approach zero curvature when the magnets are running in the same direction, i.e. magnetic field strengths are adding. This is because the bias coils (brown line) sum to produce a broad single peak with positive curvature and the curvature magnets (pink line) sum to create a very subtle two-peak distribution with negative curvature at the center. In figure 3.6, the magnets run opposite so that their magnetic field work against each other, however both the bias and curvature pairs of coils sum to create singular peaks, shown in pink and brown respectively. Thus the current must be run in opposite directions so that the curvatures add to near zero at the cost of slightly reducing the field strength.

With this in mind, if a Bitter-type magnet is constructed, care has to be considered to see if the curvature coil is correcting a bias magnet with a single broad magnetic profile or a two-peak and narrow magnetic profile. In the first case, the curvature coil needs to likely be another single-peak magnet running anti-parallel or a two-peak magnet running parallel. In the second case, the curvature coil needs to be single-peak magnet running parallel or a two-peak magnet running anti-parallel. Judging from the data in figure 3.4, Bitter type electromagnets with larger radii are more suited towards making the single-broad magnetic profiles and the smaller radii are more suited towards making the two-narrow peak magnetic profiles. We note that, in order to produce homogeneous and strong magnets, running the magnets anti-parallel is not ideal as then one magnet will subtract against the other's field strength. Thus making sure that the bias and curvature magnets have different types of magnetic field profiles is ideal for minimizing the curvature while maintaining strong field strengths. i.e. both one magnet is large in radius while the other is small.

Ideally, there should even be a threshold radius that lets a magnet transition from one type of profile to another. But due to loss of data and a lack of time, this capstone is unable to draw conclusions on where that radius lies. Additionally, the radii used are also dependent on the separation of the coils. This simulation assumed a constant distance of 2.8cm due to the vacuum chamber, but it would be interesting to see how changing this distance affects the magnetic field profile.

3.6.2 Bias-Curvature width simulation

In this simulation, we varied the geometry by increasing the width of the magnet while keeping the magnet at a constant mean radius of either $r = 4.3$ cm or 8.05 cm. The widths varied from 1.6 cm wide to 3.3 cm wide in intervals of 89 mm. Written in millimeters, the following radii were considered:

$$W = \{16, 16.9, 17.8, \dots, 33\}$$

Combinatorics simulations were not conducted with the width data because a simple conclusion could already be drawn. We observed that the magnetic profiles of larger width magnets were, up to computational error, exact multiples of the magnetic profiles of smaller width magnets. Effectively, a change in width of the magnet did not change the magnetic field profile's shape up to proportionality, as seen in figure 3.7. Given that these magnets run at various currents, that means that a magnet can simulate the magnetic field profile of magnets with smaller or larger widths, given enough variability in current. Thus the width of the magnet does not matter as much. This makes sense given our approximation that simplified the current. Since the simulations used current density, the extra width in the magnet just added a more volume for the current density to run through, in a trivial manner. Perhaps the results of this simulation would be different if we considered the more complicated $1/r$ dependent current.

Though, while our simulations doesn't place much importance on the width of the magnet, the width does matter since it changes the resistance of the magnet. Resistance is proportional to $1/\lambda$, where λ is the filling fraction of a copper layer. If the width of the magnet increases, then the volume of copper increases relative to the hole volumes which remain constant. Then since λ increases, the resistance will drop. Resistance should be kept minimal in order to lose less power due to heat dissipation, thus it is likely in our best interest to keep the width of the magnets large.

Magnetic field profile for different magnet widths

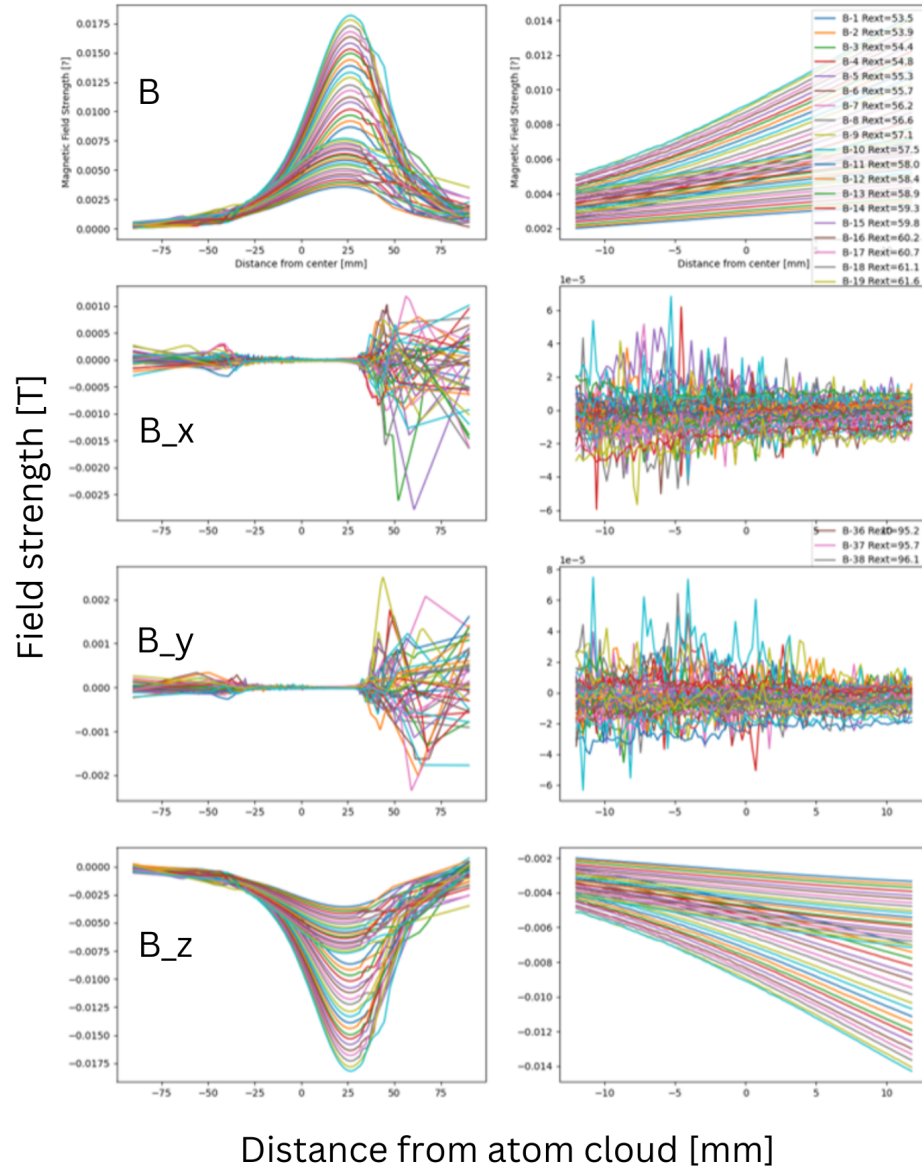


Figure 3.7: The magnetic profile of coils with 20 different widths at two different mean radii. First row is the magnetic field strength in the z direction, and the following rows are the x,y,z contributions to the field. Left column views the full range of simulated data, and the right column selects a smaller x-range view. Note that beyond $|x| > 25$ mm, the simulations get less smooth, as the mesh was more coarse past this point. This is fine given that the field at the center is the main concern. This data shows that the field profiles for magnets with different widths only differ by some factor

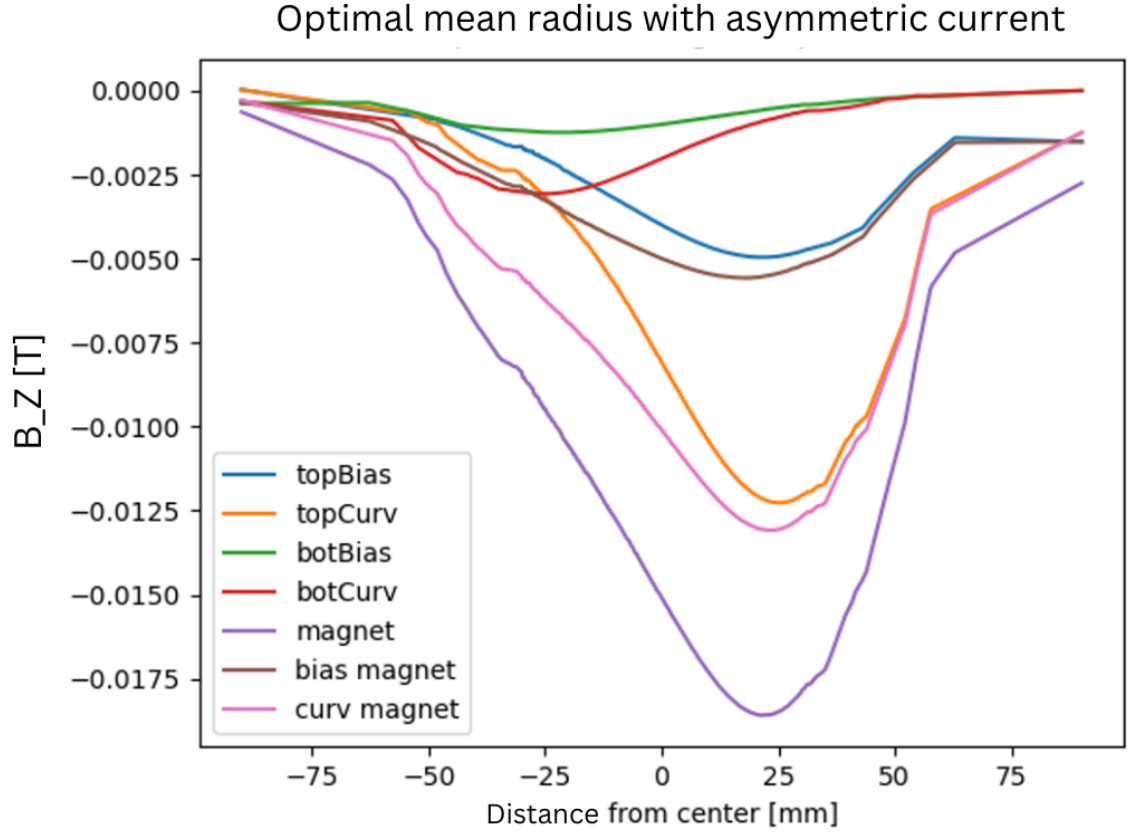


Figure 3.8: Using the ideal geometry of coils from figure 3.5, the curvature coil had to run nearly double the bias coil’s current in order to create a non-zero gradient with zero curvature at the center of the apparatus.

3.6.3 Bias-Curvature gradient simulation

As we mentioned earlier, the bias-curvature coils should be flexible enough to produce gradients in the magnetic field. This is rather easy, by running an asymmetric current across the top and bottom coils. This was feasible in any magnet configuration, given enough current. This is exemplified in figure 3.8.

CHAPTER 4

Fabrication

With the knowledge of these simulations in mind, we finalize the bias-curvature coil designs. Unfortunately, mistakes in the simulation state space and restrictions on our experimental set up does not allow us to construct the coils found in figure 3.5. But we attempt to meet its conditions by having a bias magnet with as large a mean radius as possible, to approach the helmholtz condition. Then the curvature coils are as close to the center bore so that its radius is comparably smaller than the bias. Then, the width of the coils are maximized to reduce the resistance of the coils. The final radii implemented in our construction is $R^B = (33 \text{ mm}, 47 \text{ mm})$ for the bias coils and $R^C = (18 \text{ mm}, 31 \text{ mm})$ for the curvature coils. In the following section, I make comments about the fabrication process for the bias-curvature coils.

4.1 Parts

In order to build the bias-curvature coils, we needed to procure the following parts. The schematics for all the parts can be found in the appendix A.

4.2 Water cooling with gaskets and plumbing

The bias-curvature coils will run at high currents that will generate lots of heat. We need to ensure that the water-cooling system in the design maintains a high flow rate of water. The following relationship defined how the temperature of the coil T , relates to the dissipated power P , water density ρ , water heat capacity C , and the flow rate F [10].

$$\Delta T = \frac{P}{\rho C F}$$

Given temperature's inverse relation to the flow rate of cooling, it is import to

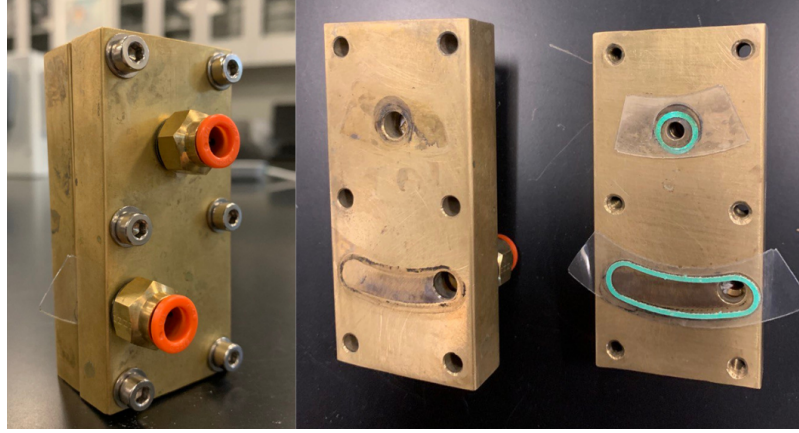


Figure 4.1: The gasket testing rig which was used to test the four types of gaskets material in the bias arc and circle configurations. The left photo shows the rig assembled while the right photo shows the inside of the two brass pieces. Water was pumped through the orange entrances and out the other wise while the gaskets were compressed by bolted brass pieces.

keep the flow rate as high as possible. Insufficient cooling can lead boiling coolant inside the electromagnet. These bubbles could cause a pressure to build up and risk damaging the electromagnet.

To test gasket efficiency, we considered several types of gaskets:

1. high strength aramid/buna-N gasket material
2. water resistance light weight paper fiber/buna-N gasket material
3. viton gasket material 0.37 mm thickness
4. viton gasket material 0.42 mm thickness

We tested all four types of gaskets and placed them under 60 bar of pressure through a test rig. To simulate the compression that the gaskets would go under inside the magnet, we tightened the test rig so that 6 bolts turned to a maximum of 1 Nm of torque. All of which worked in sealing water except for the paper fiber gaskets gaskets which immediately leaked water at the rate of 1ml/min. In the end we chose to use the aramid/buna-N gaskets because the alternative rubber gaskets left residual rubber on the test rig when removed. This wouldn't be ideal if we wanted to take apart the electromagnet and reuse the same copper layers.

Ideally it would've been better to test the gasket's leakage if the test rig included multiple layers of copper and spacer layers. However, the timeline of the project unfortunately did not allow for that.

4.3 Cutting and sanding

We used a laser cutter to cut the gaskets and polyester spacers. Other parts like the brass and copper layers had to be sent to a manufacturing shop to machine. The crucial part of this process is that the layer's thickness should be preserved as much as possible. If cut with the wrong laser cutting settings or manufactured in an less than ideal setting, the cutting edges of the material can melt so that the edge thickness of the layers slightly enlarge. Small inconsistencies in the layer thickness like this can lead to compiling errors when they're all stacked together. In the ideal case, the copper layers should all be sanded to a uniform 1mm thickness while paying special attention to the edges of the copper layers that make contact to the copper keystone layers. Then, the polyester spacers should be sanded to a uniform 0.25mm thickness, equivalent to the copper keystone pieces. If this part is sanded too thin, then the piece could be loose in the assembly. If it is still too thick, then the copper pieces could fail to connect and the magnet would not have continuous electrical connection. Lastly, we also recommend using a heat gun to heat up the polyester spacers and then pressing it flat. This step makes the assembly process a lot easier to orient and align.

4.4 Oxidation cleaning

The copper layers are exposed to a lot of air and sometimes water during the machining and sanding process. This results in the build up of the oxidized copper on the outside film of the copper layers. Because these oxide layers are high in resistance, we ought to remove them before assembling the magnet. We used an acid bath/ sonification technique to remove the grime and oxidation off of sanded copper layers. The ultrasonic waves and bubble collapse provide enough force to remove large particulates formed from the sanding process. Then, acetic acid (CH_3COOH)

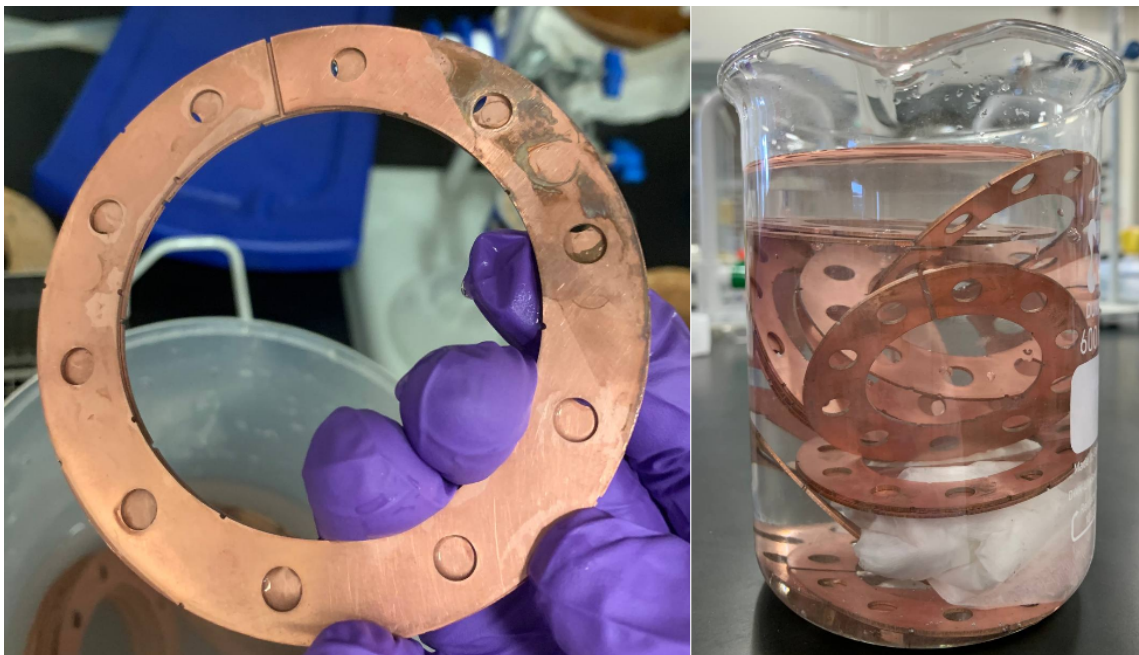
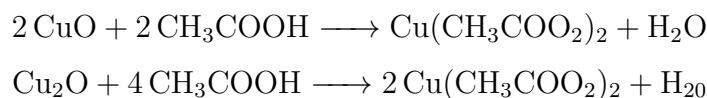


Figure 4.2: On the left is a copper layer that was partially submerged in the acid bath, resulting in a clear "before and after" comparison of copper oxidation. On the right is the diluted acetic acid bath with copper layers stacked inside to maximize surface exposure. This is the beaker that went inside the ultrasound sonifier.

can react with the copper oxide according to the following reaction [16].



To do so, we first removed any material off of the copper layers by hand. Then we placed them in a beaker with deionized water until full. We were able to deoxidize the copper layers simultaneously with the ultrasound process by mixing in 10 to 20 ml of glacial acetic acid inside the deionized water bath (make sure you don't add the water to the acid, instead of acid to the water). Then placing the beaker inside the sonification machine while setting the bath to occur at 30-50 degrees C which helps speed up the process. We recommend placing each copper layer in the bath in between 5-10 minutes. When placing the copper layers in the bath, ensure that all facets are exposed to the solution. Additionally, ensure to scrub, rinse, and dry the copper layers after wards, while taking special care to the sections of the coil in contact with the keystones.

4.5 Assembly

Figures 4.3 and 4.4 present some of the first few layers and end product of the assembly process, to give an idea of how the pieces work in conjunction to carry current helically. After the assembly of the curvature coils, it is important to align and compress it before assembling the bias coils.

Alignment is the process of making sure that the layers are flat with one another and well organized. During the layering process, all the layers are aligned to the titanium rods that stick out in the figure. But that is not enough, after all the layers are stacked, extra care must be taken to ensure that the layers were stacked correctly, because this cannot be changed later after compression without laboriously decompressing the rig. The design features small ridges that line the rim of the layers so that each layer can be aligned with the others. Additionally, one should also check that the spacer layers don't have an overlap between the copper keystone and the polyester spacers. Otherwise, no amount of compression will get rid of bad conductivity issues. The alignment process will likely be smoother if care was taken to ensure all the parts, especially the gaskets and polyester spacers, are flat.

Once aligned, one can start tightening the bolts. Note that the bolts used in the end design are much shorter than those shown in the assembly picture. This is because the bolts should have a nut that is flush with the surface of the brass, as there is no space for them to go further than there when these are embedded in the cold-atom experiment. We found that 75 mm is a sufficient length for the rods to lengthen these rods. As seen in figure 4.5, the rod that is counter clockwise from slit is made of brass to allow for electrical transfer to the brass layer. This brass rod should be longer like 90mm, so that electrical connections can be attached on the backside. The bolts were tightened on the brass side with a gasket, brass washer, and brass nut. The other side used a titanium nut. The gaskets and washers are there to keep water sealed while water is pumped in to the apparatus through another hole in the cap. The compression process applies a lot of force on the coils and condensed our bias coil by 2.27 mm and our curvature coil by 3.09 mm. However, we emphasize that thread failure was a persistent issue throughout compression, as some threads strip and fail to tighten any further, at which point, the thread had to be replaced. This was a significant issue with the brass rods, given brass's softness as a metal. Adequate compression is defined by two factors: electrical connection and water tightness. Based on intended fabrication lengths, the coils should be compressible to

a theoretical minimum of 39.1 mm. But the fabricated curvature coil was longer than this height by 4 mm, and the bias coils were slightly further away. As you approach this length, the resistance should drop continuously with each turn of the nuts. The theoretical minimum resistance for both coils is around $12\text{ m}\Omega$ so one can check using a multimeter whether the resistance is gradually getting closer to this value. Given our magnet compression, we were able to get as close as $143\text{ m}\Omega$ for the bias coils and $85\text{ m}\Omega$ for the curvature coils. We note that the water tightness of the coils was not tested, which may give incentive to improve the compression and alignment of the magnet if the magnet leaks, even if the current resistance measurements are sufficient for experiments.

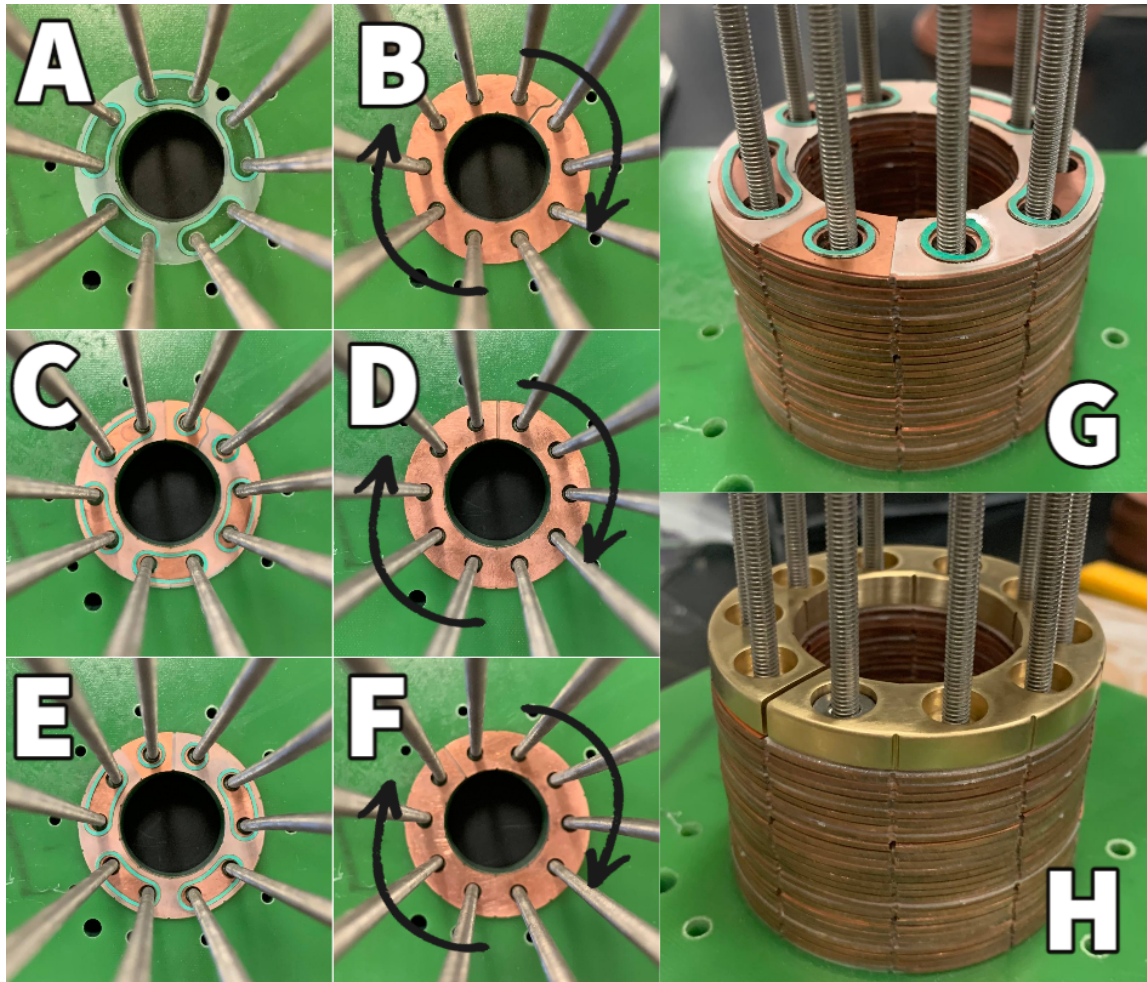


Figure 4.3: A collection of images featuring the assembly process for the curvature coil. Figures A-F feature the first few layers when building from the mounting cap towards the brass cap. The first layer A has a special spacer designed to let a copper rod make contact with the first layer in picture B. Note that the S-shaped slit in layer B was placed in the wrong direction but was corrected in picture C, as can be seen with the S-shaped copper keystone. The slits move in a counter clockwise direction as seen in B,D, and F. This allows the current to flow in a clockwise direction, as denoted by the black arrows. Notice the green gaskets placed to accommodate the water cooling in the spacers of layers A,C,and E. picture G is the last layer of spacers before placing the brass cap in picture H.

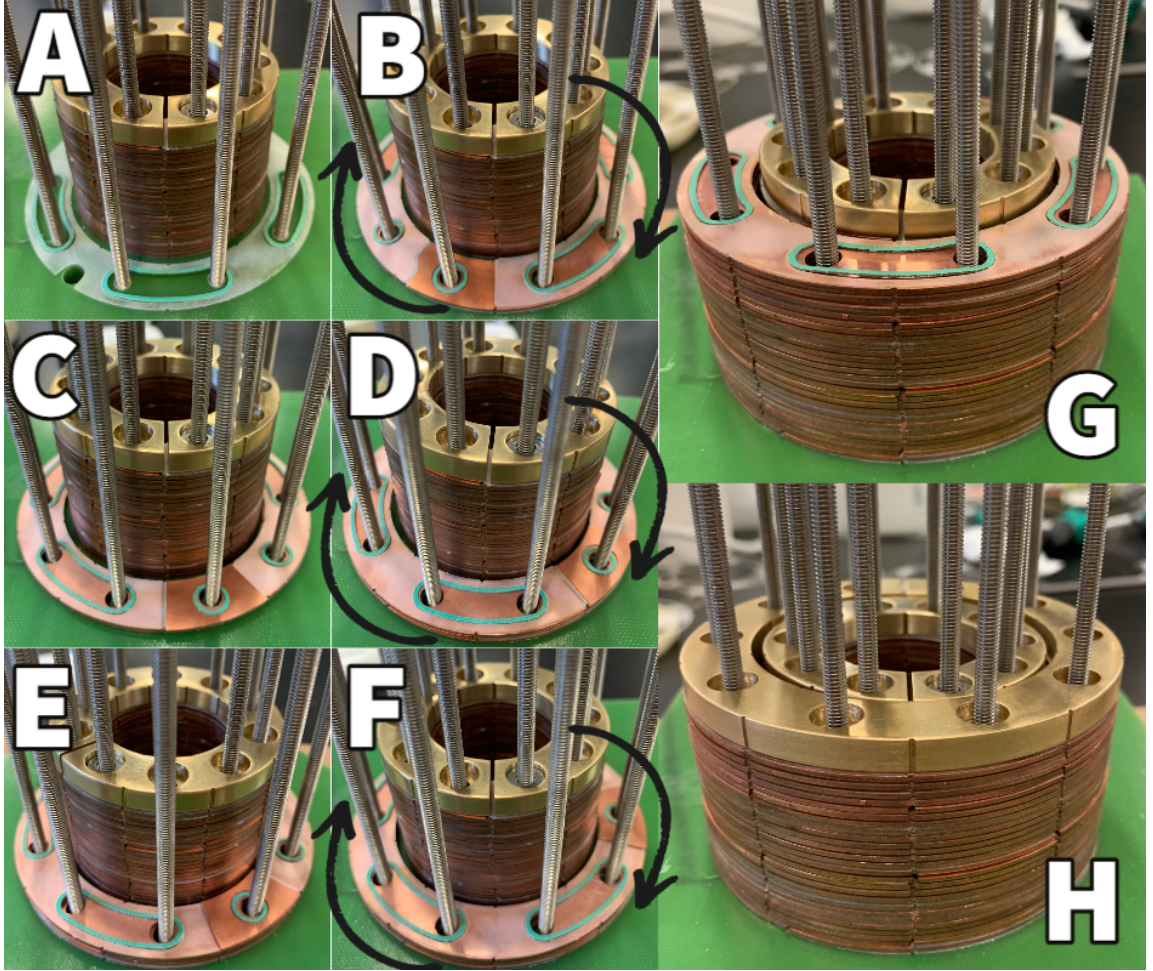


Figure 4.4: A collection of images featuring the assembly process for the bias coils. Similar to the design of the curvature coils except the starting spacer in layer A is starts on the opposite side, as denoted by the whole in the cap. This design is in place to hopefully nullify the magnetic contribution of kestones in the curvature coil moving current upwards. Since the bias and curvature coils have kestones in opposite sides, any magnetic field generated in the polar directions should be minimized. there are many types of magnets. Like before, the current can still be considered as flowing clockwise from the first. Note that assembly of the bias coil should not be done until after the curvature coil is compressed, unlike shown in this image.

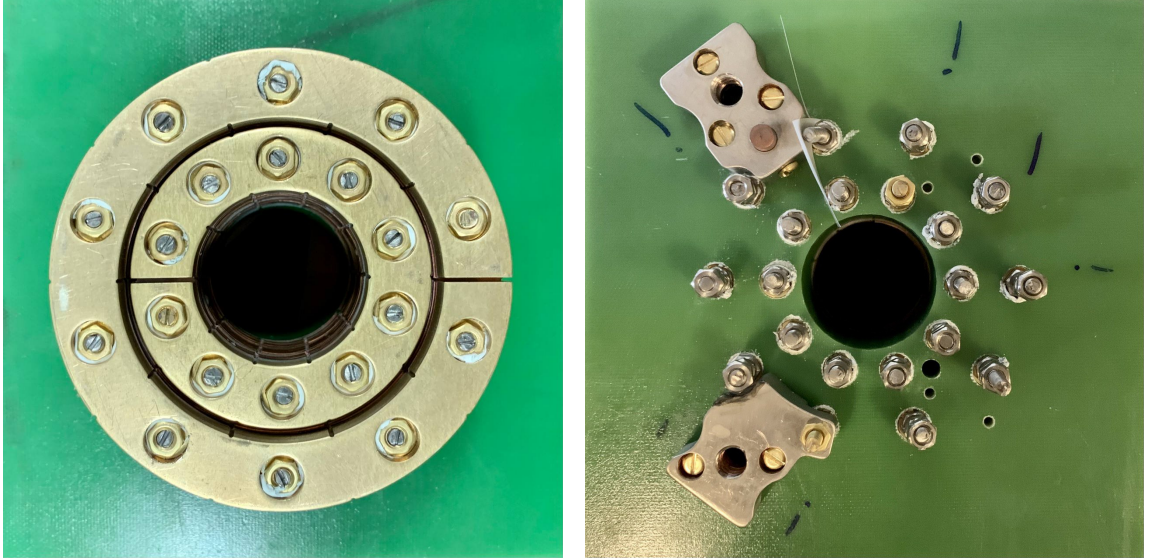


Figure 4.5: The left figure a) The brass side of the bias-curvature coils, mounted on a fiber glass testing rig. This is the side that is closest to the atom cloud. Notice that most of the holes have titanium rods fastening the magnet while the hole counter clockwise from the slit has a brass rod for electrical connection. The right figure b) The fiber glass side of the temporary cap rig. This is the side that connects the magnet to the pumps and the electrical generator. The nuts on this side are where most of the compression happens. Also pictured are the nickel plated copper lug holders and lug screws for the bias coil. Notice that some of the titanium rods bent during the tightening process, another problem that was similar to the thread failure. We put insulting paper in between that and the lug holder to prevent the short circuits.

CHAPTER 5

Testing

5.1 Field measurements

In the absence of the fiberglass cap for the apparatus, we had to test and measure the magnetic field profiles of the bias-curvature coils on a test rig.

We remind that the measurements should be completed after one of the coils is assembled and compressed, since the the bias coils get in the way of aligning and compressing the curvature coils. This is because the curvature coils needed to be checked for sufficient compression with the testing set up. If not compressed enough, the current generator couldn't generate any substantial potential difference across the magnet due to high resistance. The test rig could not support water cooling like the actual cap can so we could only run the magnets at low currents to prevent heat up of the magnets. For the curvature coils with resistance 85 mΩs, we could didn't go higher than 1 ampere. The bias with resistance 143 mΩ didn't go higher than 0.2 amperes. Using a lake shore magnetometer, a test rig was designed to measure the magnetic field along the axial direction. Then using that data, we fit it to a finite continuous solution for a solenoid [5.1.1](#)

$$B_z = \frac{\mu_0 N I}{2} \left(\frac{l/2 - (z - c)}{l \sqrt{R^2 + (l/2 - (z - c))^2}} + \frac{l/2 + (z - c)}{l \sqrt{R^2 + (l/2 + (z - c))^2}} \right) \quad (5.1.1)$$

Here, B_z is the magnetic field strength in the axial direction, N is the number of turns in a magnet, I is the current, l is the length of the magnet, R is the radius of the magnet, and c is a constant to define where the peak field is. While this equation is only valid for solenoids with infinitely thin wires, we actually find that it can still be used as a model to fit our data to, even though the constants don't necessarily match up with real measurements.

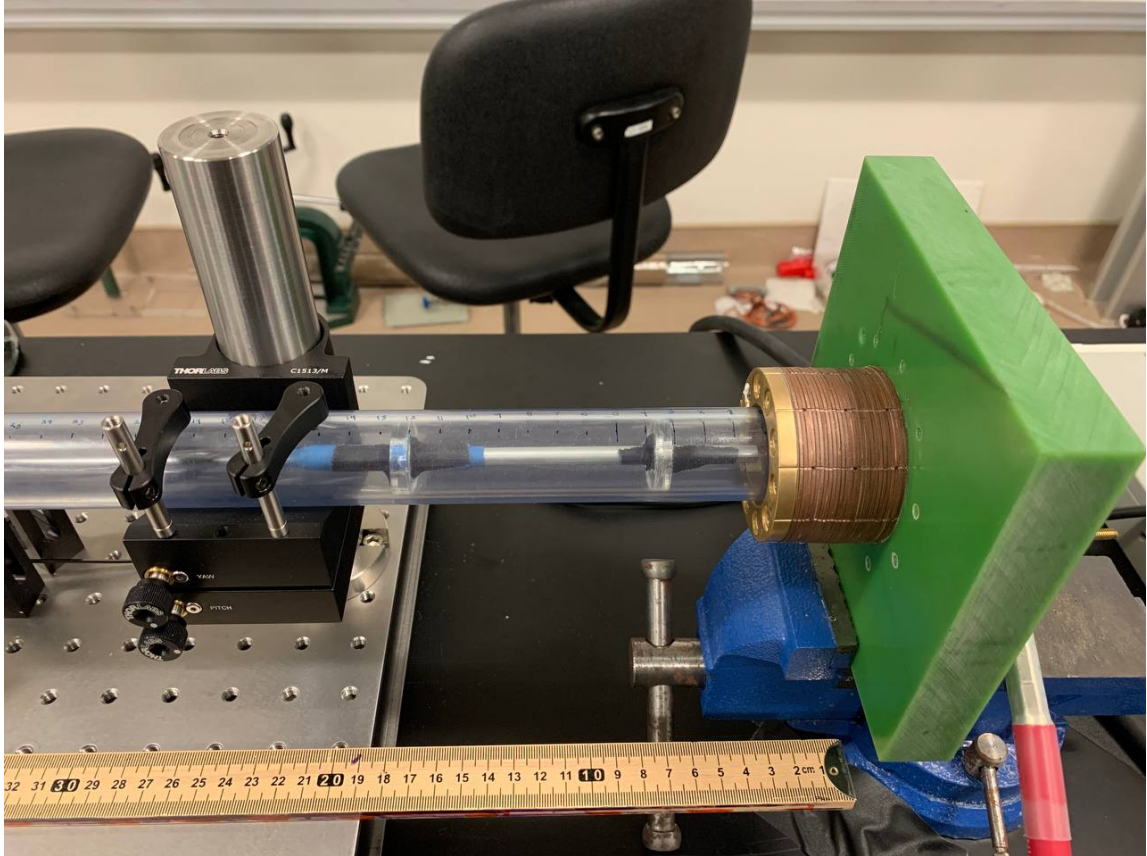


Figure 5.1: The set up to measure the magnetic field of the curvature coils. The tube with centimeter markings shown in the figure contains the magnetometer and was used to measure the field as the magnetometer moved into the brass cap. The ruler was used for measurements where the tube went further away from the magnet. The magnetometer was mounted on an optical table for stability and the curvature coils were mounted on the green fiberglass.

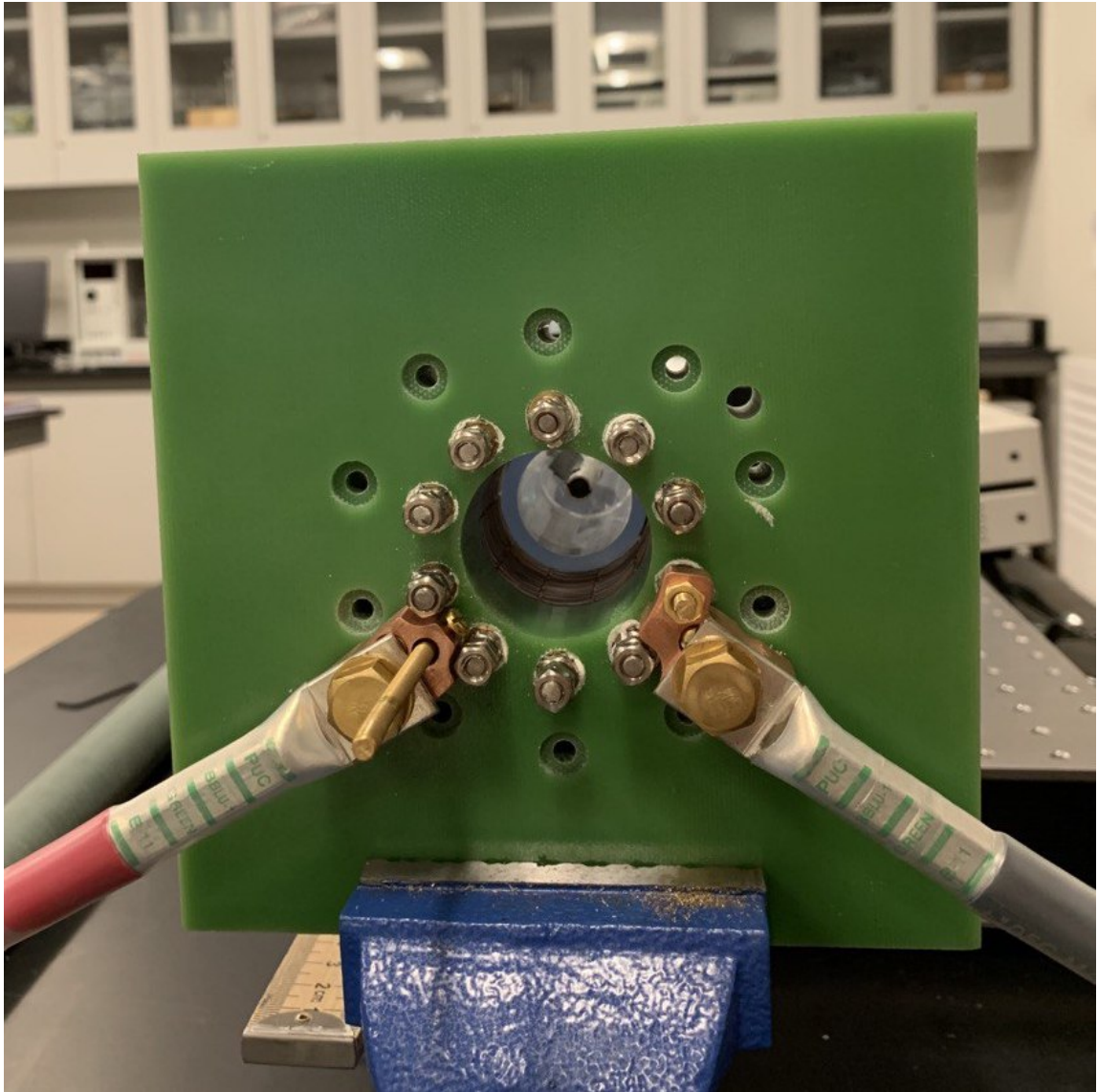


Figure 5.2: This picture shows how the electrical connections form the current generator attached to the magnet. The current would attach, via extra copper lugs parts, to the brass rod on the back side of the magnet. Then the brass rod connected to the brass cap on the other side which allowed current to flow back to the cap through the copper layers. Then the foundational copper layer was attached to the copper rod that is pushed with pressure against the layer. This copper rod then connects to the lug.

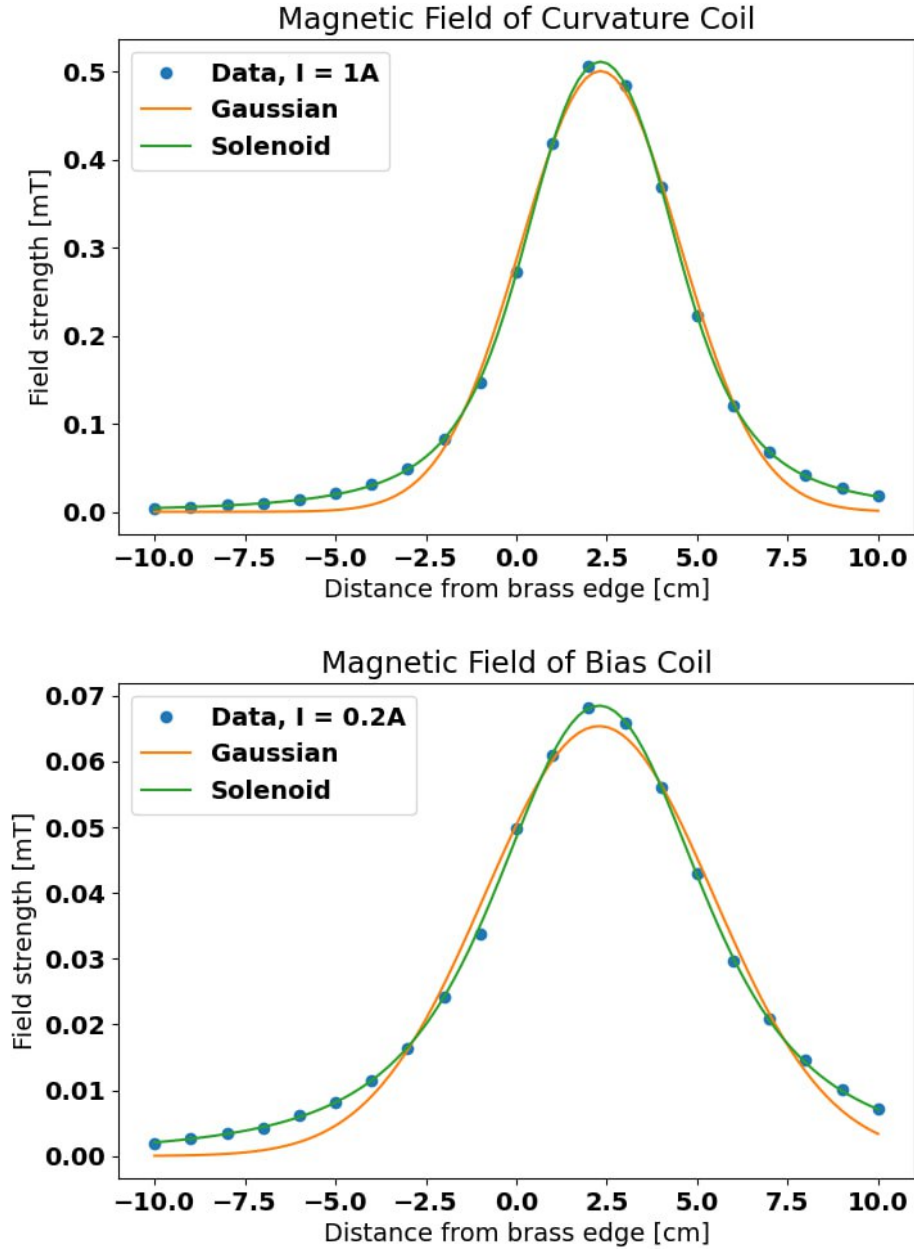


Figure 5.3: Data measured along the axial direction for curvature and bias magnets. Note that the data was measured using two different currents as the bias coil had nearly double the resistance of the curvature coils. To avoid heating up the coils, we were cautiously applied 1 A current to the curvature coil and a 0.2 A current to the bias coils. Nonetheless, both profiles should scale linearly with current. The finite solenoid best fit had very low uncertainties on the parameters that define the equation as seen in table 5.1. The Gaussian fit did not fit as well.

fitting parameter	curv fit	bias fit	measured curv	measured bias
$\mu_0 NI/2$	1.50 mT/cm	0.289 mT/cm	NA	NA
l	3.59 cm	3.55 cm	4.1 ± 0.2 cm	4.1 ± 0.2 cm
R	2.32 cm	3.84 cm	2.38 ± 0.03 cm	4.02 ± 0.01 cm
c	2.33 cm	2.29 cm	NA	NA

Table 5.1: fitted parameters. the $\mu_0 NI/2$ parameter estimation doesn't make sense in our set up so we couldn't measure the values. Instead, it should just be treated as a proportionality constant that affects the volume that the current density is multiplied against in the discrete approximation of biot-savart's law 3.2.1. Here, the length of and radius of the constructed bias and curvature coils are also reported to compare to the fit parameters. The radius measurement is the mean radius of the bias and curvature coils.

As seen in figure 5.3, the data is best fit by the solenoid expression in equation 5.1.1, rather than other equations like the Gaussian shape. Through this fitting, the fitting algorithm predicted the constants found in table 5.1. The estimated values align somewhat closely with measured values which tells us that equation 5.1.1 can predict, up to some proportionality constant, the magnetic profile some Bitter-type electromagnet, but measuring what the constants for c , l , and r for a physical Bitter-magnet is unclear given its geometry. The discrepancies in between the fit and the measurements are likely due to factors like different thicknesses of layers, and the finite width of the electromagnet. It could also be caused by an uneven current distribution in the magnet, since the current density in a real magnet would run $\propto 1/r$. The discrepancy tells us that real magnets then have an effective length and radius, if it were modeled using the finite solenoid equation. Thus the equation is better used to simply interpolate data which is useful for describing the field strength, gradient, and curvature of the magnet along its z direction, instead of predicting the field profile a bias-curvature coil of some dimensions l , r , and c . For example, the interpolation data tells us that the maximum field strength occurs 2.32 cm into the magnet when measured from the brass cap. Alternatively, the interpolation allows us to compute the magnetic field strength, gradient, and curvature at any point around on the magnet's axial direction.

5.2 Constructing a Bias-Curvature coil

It is not necessary to construct the second pair of bias and curvature coils because the apparatus is symmetric. Like how we did during the simulation phase, we use the linearity of magnetic fields to predict the full operational capacity of the bias-curvature coils.

5.2.1 Validating linearity

While it is well known with theory that magnetic fields add linearly, we need to validate whether the magnetic field of the Bitter-type electromagnets indeed scale linearly with current. This was an assumption we made during the simulation phase of the project that assumes that the current runs constantly and trivially in the $\hat{\phi}$ direction. In reality the current is $\propto 1/r$ and also has components in the z direction due to the spacer layers and possible electrical shortages to the titanium rods. To test this assumption, we ran the magnetic field profiles of the curvature coils at 1 A and 2 A and found that the 2 A magnetic field profile is on average, 2.004 times strong than the 1 A magnetic field profile. This gives us confidence to just multiply magnetic field profiles by some factor to arrive at magnetic fields for other currents.

5.2.2 Constructing an operational profile

We would like to construct an operational profile where given some field requirements, i.e. 832 gauss in strength and zero curvature, we can know what configuration of currents would be used. Seeing as we validated our use of linearity, we can use the magnetic field profile at 1 A and the 0 magnetic field with 0 A to construct linear equations relating the field strength, gradient, and curvature.

$$\begin{aligned}
 B_B &= 0.3032 I_B \text{ [mT]} = 3.032 I_B \text{ [G]} \\
 B_C &= 0.1457 I_C \text{ [mT]} = 1.457 I_C \text{ [G]} \\
 \frac{\partial^2 B_B}{\partial z^2} &= 328 I_B \text{ [mG/cm}^2\text{]} \\
 \frac{\partial^2 B_C}{\partial z^2} &= 554 I_C \text{ [mG/cm}^2\text{]} \\
 \frac{\partial B_B}{\partial z} &= 2.22 * 10^{-15} I_B \text{ [mG/cm]} \approx 0 \\
 \frac{\partial B_C}{\partial z} &= -1.68 * 10^{-15} I_C \text{ [mG/cm]} \approx 0
 \end{aligned}$$

A consequence of this result means that we can find a unique set currents for the bias and curvature coils to meet some operational profile. For some target field $B = B_B + B_C$ in units of gauss and some target curvature $\frac{\partial^2 B}{\partial z^2} = \frac{\partial^2 B_B}{\partial z^2} + \frac{\partial^2 B_C}{\partial z^2}$ in units of mG/cm²,

$$\begin{aligned}
 I_B &= 0.461 B - 0.00121 \frac{\partial^2 B}{\partial z^2} \text{ [A]} \\
 I_C &= -0.273 B - 0.002527 \frac{\partial^2 B}{\partial z^2} \text{ [A]}
 \end{aligned}$$

Given this operational profile, our target of 832 gauss with 0 curvature can be obtained with both bias coils operating with 379 amps of current while the curvature coils running with -224 amps of current, ass seen in figure 5.4.

If we are to fix curvature to zero and sweep the magnetic field strength, we observe a relation such that $I_C/I_B = -0.592$

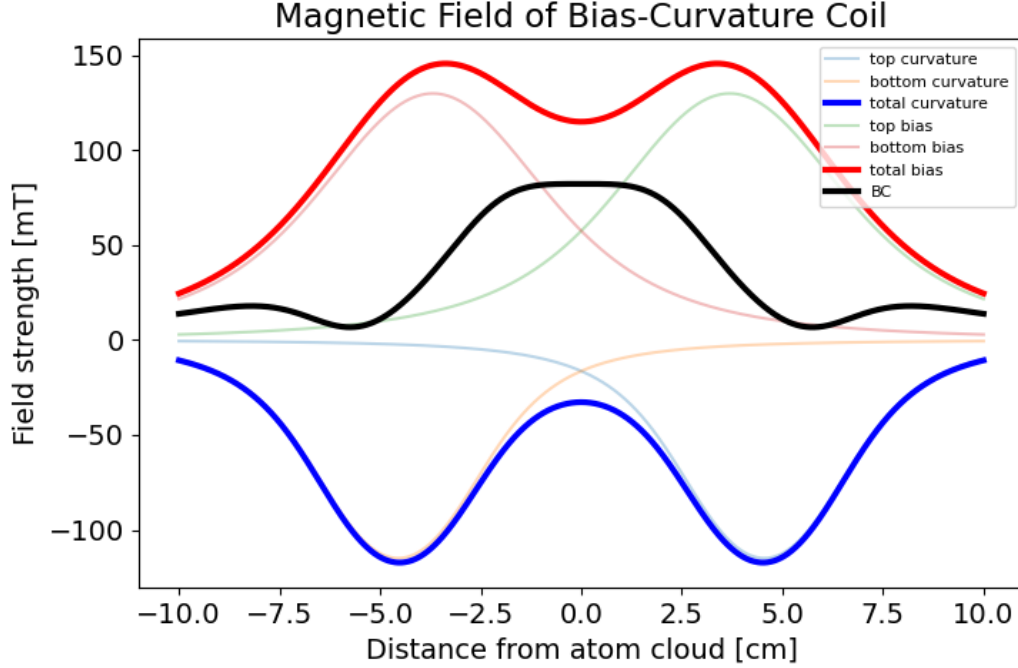


Figure 5.4: Sample bias-curvature magnetic profile, created from reflecting data from figure 5.3 to simulate four electromagnet ensembles from two. Both coils are normalized to have a current of one amp, but running in opposite directions. Using Savitzky-Golay filtering, the curvature of the magnetic field here is estimated to be $\approx -0.0135 \text{ mG/cm}^2$, with the bias and curvature coils individually producing a curvature of 124291 mG/cm^2 in opposite signs. Then the peak field strength is about 832 gauss, the lithium-6 Feshbach resonance.

5.3 Thermal properties

The resistance of the coils can be estimated with

$$R = \frac{2\pi N \rho_0}{d \lambda \ln(R_{out}/R_{in})}$$

$$\lambda \approx 1 - \frac{10\pi 6^2}{\pi(R_{out}^2 - R_{in}^2)}$$

where $N = 26$ layer of copper, $\rho_0 = 1.7 * 10^{-8} \Omega$ is the resistivity of copper, $d = 1$ mm is the thickness of copper. Lambda is the filling fraction which factors in how the water-cooling holes don't carry current. The bias and curvature coils have different radii, so $\lambda_B = 0.68$, $\lambda_C = 0.43$, and then the resistances for both coils are predicted to be around 12 m Ω . This differs largely from our measured resistance as the one bias electromagnet had 143 m Ω of resistance and the curvature had 85 m Ω . This is likely because of insufficient compression for the system. Before compression, the bias-curvature coils had resistances on the order of a mega ohm. But as compression tightened the coils, the copper spacers gain more surface contact to each layer, reducing the resistance. As noted in the fabrication section, we did not compress the magnet to its theoretical maximum, thus our measurements were likely conducted at sub-optimal resistances. However, we continued with these resistances because further compressing the magnet became harder without bolt threads failing. Perhaps even still, the sanding on the spacers of copper wasn't enough, or the gasket and spacers weren't aligned properly. Such issues would cause inconsistencies in thickness that were incompressible, leading to copper spacers being poorly connected to the copper layers. But for the purposes of analysis, we will continue quoting thermal properties assuming the ideal and non-ideal cases for resistance.

The temperature rise in the water cooling system follows the following relation: $\Delta T = P/mC$ where the mass flow rate $m = F * 1000/60$ for a flow rate in liters per minute, and the heat capacity of water is $C = 4.186$. We assume a flow rate of 3.8 which was one of the test flow rates used in [10]. Then, running the 832 gauss and 0 curvature operational profile suggests a 78°C and 16°C increase in bias and curvature coil respectively. However, if we forgo our measured resistance and use the theoretical optimum resistance, we arrive at bias and curvature coil temperature change of 7°C and 2°C. Clearly, reducing this resistance is very important to the effectiveness and safety of these coils. Additionally, our lab set up has a maximum of 6.0 l/m flow rate, when the pumping set up is not hooked up to the magnet.

Further work in this project would quantify how much the bias-curvature magnets reduce the flow rate. For this analysis, we hope that it does not reduce it more than 3.8 l/min, otherwise many of the temperature estimations here would have to be reconsidered.

$$\Delta T = \frac{(I^2 R)}{mC}$$

Given the difficulty in compressing the magnet, we note down the operational threshold for the electromagnet. Like in [10], we cite 100°C as our maximum temperature increase, at which point the water would begin to build up pressure inside the electromagnet's cooling ducts, potentially causing cracks to burst open. Assuming a coolant flow rate of 3.8 l/min, this threshold would occur at about 26kW of heat dissipation. At optimal resistance, this occurs when running a current of $I = 1.48kA$. At our measured resistance, the bias coils with a resistance of 143 mΩ fails to operate at $I = 430A$, and a curvature coil resistance of 85 mΩ fails to operate at 558 amps. These maximal currents means that we can create a maximum bias field strength of 1300 gauss. And in order to correct for the magnetic curvature to zero, the curvature coil would reduce the field strength and end up with a bias-curvature coil with around 1000 gauss and 0 curvature. Fortunately, this maximum operational level surpasses our hope of producing field on the order of 832 gauss and zero curvature case.

We also note the currents needed to produce 832 G field with 200 mG/cm² curvature case only different from the 832 gauss and zero curvature case by 0.2 amps in the bias and 0.5 amps in the curvature coils. Thus, in order to map the inhomogeneity $\tilde{h}/2\pi < 2$ region shown in figure 1.3, we also need precise control over the current of the apparatus.

5.4 Concluding remarks

Through the efforts of this capstone, we edge closer to a fully realized quantum simulator with ultra cold lithium-6 in Professor Ben Olsen's Olsenlab.

We build upon the research presented by Smale et al. and unlock the potential of realizing a new domain of phases in quantum matter. Through simulation and design-testing we arrived at a "bias-curvature" electromagnet design capable of not

only producing a strong magnetic field but also zero magnetic curvature, by using a "bias" Bitter-type electromagnet to provide a strong magnetic field and a concentric but smaller "curvature" Bitter-type electromagnet to correct the magnetic curvature. Given this magnetic field capability, we can proceed with building the full bias-coil geometry and embedding it in an integrated cold-atoms experiment, with laser cooling, zeeman slowing, magneto-optical trapping of cold lithium atoms.

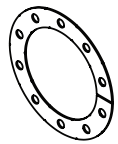
We used a finite element method simulation in order to consider how the variable geometries of a bias-curvature coil could affect its magnetic field capabilities. This was possible by first generating a 3D mesh of many differently designed bias-curvature coils in GMSH, and then secondly using FreeFEM++ to solve the weak formulation of Maxwell's equations for magnetic fields, and thirdly using linearity and combinatorics tricks to find the best design for a bias-curvature coil. Our computational simulation concludes that in order for bias-curvature coils to be effective, one coil should be as close as possible to the helmholtz condition as possible, where mean radius is used to accommodate for the thickness of the coils. Given that the radii and magnet separation distance determines whether the magnetic profile is a single broad profile or a two peak profile, we suggest that the correcting coil would should have a different type of magnetic profile shape from the bias, so that it can run in parallel direction such that magnetic field strength add but curvatures cancel. It is possible to construct bias-curvature coils where the bias and curvature have the same magnetic profile type, however it requires that the magnetic curvatures cancel at a faster rate per ampere than the rate at which magnetic fields cancel. This type of set up works, however ultimately requires more current and is less optimal.

Because of the restraints on geometry imposed by a vacuum chamber for cold atoms, we did not fabricate the optimal design of a bias-curvature coil. We constructed the second variety of bias-curvature coils that requires more current than the ideal. With a bias coil radii of $R^B = (33 \text{ mm}, 47 \text{ mm})$ and a curvature radii $R^C = (18 \text{ mm}, 31 \text{ mm})$, we were able to experimentally show the curvature-canceling properties of this magnet. We measure the bias-curvature coil's magnetic field profile for 1 A; and the linear relation of the magnetic field and current allow us predict the field output at higher currents. Further more, the symmetry of the bottom and curvature coils allow us the predict the full bias-curvature field profile without constructing the top and bottom bias-curvature ensembles. Given those measurements, we estimate the magnetic field strength, gradient, and curvature at the center per unit of current. Using those, we solve some linear equations to arrive at an operational profile of cur-

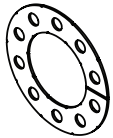
rent configurations that lead to a particular field strength and particular magnetic curvature. Using this operational profile and an assumed rate of 3.8 l/min coolant flow, we can estimate that our magnet can produce a maximum of 1000 gauss with near zero magnetic curvature, before heating becomes too much and the coolant boils inside the magnet's cooling ducts. This is of course assumes that the flow rate of 3.8 l/min is attainable with the magnet which is yet to be tested. Regardless, this gives us confidence that the design can attain our desired output of 832 gauss and 0 magnetic curvature, for lithium-6 cold atom experiments.

Our design is not optimal, but is sufficient for cold atom experiments with lithium-6. There is still room for improvements that do not require large design changes, like compressing the magnet further to approach the theoretical resistance of $12\text{ m}\Omega$, which is up to 12 times smaller than our bias' measured resistance. Then, the magnet should be built on the actual cap so that the water-flow could be tested. Previous experiments with Bitter-type magnets of this design struggle with getting adequate water sealing, given the many gaskets that need to be aligned and compressed. This capstone also does not experimentally explore the possibility of magnetic profiles with constant gradient, which is another interesting magnetic profile to have for cold quantum gas experiments. On the simulation side of things, we regret not restraining the simulations to the appropriate dimensions to better optimize the geometry for our quantum gas experiment. But our exploration leaves open several research questions for further work such as: At what radius is the helmholtz condition satisfied with a Bitter-type electromagnet with finite thickness? Within the domain of bias-curvature magnets that have field strengths that work against each other but still produce zero curvature, how can we minimize the rate at which field strength's cancel per unit of current. Or to what degree does our simplification of current matter in our simulations, and could simulations with better memory usage algorithms tackle a more realistic but complicated mesh? We leave it to further work in the Olsen lab or beyond to tackle these questions.

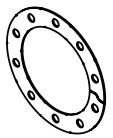
Appendix A: Bias-Curvature Coils-parts list



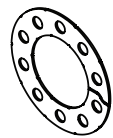
Copper arc (config-BiasA) x 50



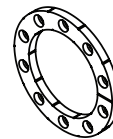
Copper arc (config-CurvA) x 50



Copper arc (config-BiasB) x 2



Copper arc (config-CurvB) x 2



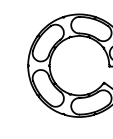
Brass arc (config-Bias) x 2



Brass arc (config-Curv) x 2



Spacer arc (config-BiasA) x 50



Spacer arc (config-CurvA) x 50



Spacer arc (config-BiasB) x 2



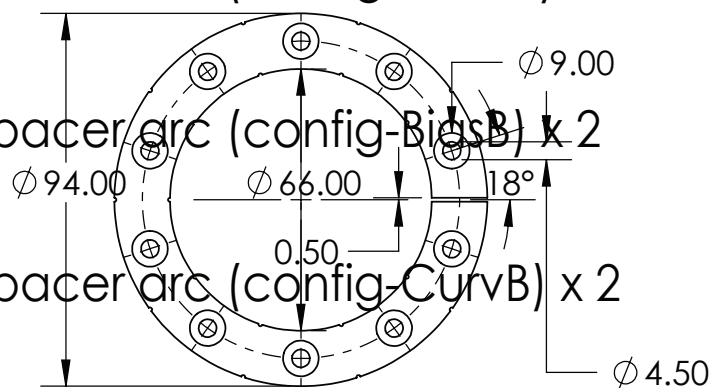
Spacer arc (config-CurvB) x 2



Spacer arc (config-BiasC) x 2



Spacer arc (config-CurvC) x 2



Appendix B: Below is sample code used for GMSH mesh generation.

```
//The following code is written in the GMSH software's script for defining a mesh, using a mix of the default geometry
//kernal and the open cascade geometry kernal.
//This code implements the full bias curvature geometry, and was modified to produce the simplified geomerty
//simulations. That was a simple matter of removing some of the lines.

//////////
///Optimization///
//////////
Mesh.Optimize = 1;
Mesh.MeshSizeFromPoints = 1;
Mesh.MeshSizeFromCurvature = 0;
Mesh.MeshSizeExtendFromBoundary = 1;
Mesh.MeshSizeMin = 0.001;
Mesh.MeshSizeMax = 1;
Mesh.MeshSizeFactor = 1;

//////////
///Parameters///
//////////
DefineConstant
[
  R = {0.1600, Name "Geometry/01 Infinite boundary radius"},
  LM = {0.009, Name "Geometry/02 Brass layer length"},
  RBext = {0.0720, Min 0.0560, Max 0.0940, Step 1, Name "Geometry/03 Bias Magnet external radius"},
  RBint = {0.0560, Name "Geometry/04 Bias Magnet internal radius"},
  RCext = {0.0330, Name "Geometry/05 Curv Magnet external radius"},
  RCint = {0.0220, Name "Geometry/06 Curv Magnet internal radius"},
  phiSlit = {0.005, Name "Geometry/07 Slit angle for magnets"},
  dSlit = { 0.01, Name "Geometry/08 Slit in linear distance measurement" },
  sepBC = {0.038, Name "Geometry/09 Seperation of BC coils"},
  WaterHoleR = { 0.0035, Name "Geometry/10 Radius of cooling holes"},
  HoleNumber = { 8, Name "Geometry/11 Number of holes on a layer of magnet"},
  HoleCenterB = { RBint+(RBext-RBint)/2, Name "Geometry/12 Radius of cooling holes' center bias coils"},
  HoleCenterC = { RCint+(RCext-RCint)/2, Name "Geometry/13 Radius of cooling holes' center curvature coils"},
  ZProbe = {0, Name "Geometry/14 Boolean: Probing Z direction"},
  XProbe = {0, Name "Geometry/15 Boolean: Probing X direction"},
  spacerHeight = {0.001, Name "Geometry/16 Height of spacer"},
  copperHeight = {0.001, Name "Geometry/17 Height of copper layer"},
  numLayers = {26, Name "Geometry/18 Number of copper layers"},
  h = {0.07, Name "Mesh/01 Characteristic length for boundary"},
  hM = {0.05, Name "Mesh/02 Characteristic length for magnet"},
  hP = {700, Name "Mesh/03 Characteristic length of probing region"}
];

//Defining Functions//
SetFactory("OpenCASCADE");

//MACRO FOR CREATING LAYERS OF BIAS CURVATURE COILS
Macro CreateLayer
  //////////
  ///Making Cylinders///
  //////////
  CylVol = newv;
  Cylinder(CylVol+0) = {params[0], //X
    params[1], //Y
    params[2], //Z
    0, //X vector
    0, //Y vector
    params[3], //Z vector
    params[4]}; //INTERIOR radius
  Cylinder(CylVol+1) = {params[0], //X
    params[1], //Y
    params[2], //Z
    0, //X vector
    0, //Y vector
    params[3], //Z vector
    params[5]}; //EXTERIOR radius
  //////////
  ///Making slicing rectangles//
```

```

////////////////////////////////////
SlitVol = newv;
Box(SlitVol+0) = {params[4]-0.005, //X
                 -params[8]/2, //Y
                 params[2], //Z
                 params[5]-params[4]+0.005*2, //X length
                 params[8]*1.02, //Y length
                 params[3]}; //Z length

////////////////////////////////////
//Making holes//
////////////////////////////////////
HoleVol = newv;
HoleCenter = params[4]+(params[5]-params[4])/2; //Calculating radius of hole centers
WaterHoleR = (params[5]-params[4])*0.25; // Makes hole 50% of thickness in diameter
For iter In {0:params[7]-1:1}
    Cylinder(HoleVol+iter) = {HoleCenter*Cos(2*Pi/params[7]/2 + 2*Pi/params[7]*iter), //X
                             HoleCenter*Sin(2*Pi/params[7]/2 + 2*Pi/params[7]*iter), //Y
                             params[2], //Z
                             0, //X vector
                             0, //Y vector
                             params[3], //Z vector
                             WaterHoleR}; //Hole Radius

EndFor
////////////////////////////////////
//subtracting holes from layer//
////////////////////////////////////
LayerVol = newv;
BooleanDifference(LayerVol+0) = { Volume{CylVol+1}; Delete; }{ Volume{CylVol+0, SlitVol+0,
                             HoleVol+0:HoleVol+params[7]-1}; Delete; };

//Rotating layer//
Rotate{ {0,0,1}, {0,0,0}, params[9] }{ Volume{LayerVol+0}; } //bias rotator

//Add to list
Magnet[] += LayerVol+0;
MeshSize{ PointsOf{ Volume{LayerVol+0};} } = 0.005;

If (params[10] == 1)
    Bias[] += LayerVol+0;
EndIf
If (params[10] == 2)
    Curv[] += LayerVol+0;
EndIf

Return

////////////////////////////////////
//Global variables//
////////////////////////////////////
SetFactory("OpenCASCADE");
Wall[] = {};
Bias[] = {};
Curv[] = {};
Magnet[] = {};

////////////////////////////////////
//bounding box//
////////////////////////////////////
bounds = newv;
Sphere(bounds+0) = {0,0,0,R};
If (XProbe)
    Rotate { {0, 1, 0}, {0, 0, 0}, -Pi/2.} {
        Volume{bounds+0};
    }
EndIf

////////////////////////////////////
//Magnet//Magnet//Magnet//Magnet//
////////////////////////////////////

//TOP BRASS BIAS COIL

```

```

params[] = {0, //Params 0 : X position
0, //Params 1 : Y position
0, //Params 2 : Z position
LM, //Params 3 : Layer Height
R Bint, //Params 4 : Inner Radius
R Bext, //Params 5 : Exterior Radius
Pi, //Params 6 : Slit Angle
8, //Params 7 : number of Holes
phiSlit, //Params 8 : slit width
0, //Params 9 : slit direction
1}; //Params 10 : type of magnet 1 = BIAS, 2 = CURVATURE

Call CreateLayer;

//TOP COPPER BIAS COIL
For i In {1:21}
    params[] = {0, //Params 0 : X position
0, //Params 1 : Y position
LM+spacerHeight*((2*i)-1), //Params 2 : Z position
copperHeight, //Params 3 : Layer Height
R Bint, //Params 4 : Inner Radius
R Bext, //Params 5 : Exterior Radius
Pi, //Params 6 : Slit Angle
8, //Params 7 : number of Holes
phiSlit, //Params 8 : slit width
2*Pi/HoleNumber*(i), //Params 9 : slit direction
1}; //Params 10 : type of magnet 1 = BIAS, 2 = CURVATURE

    Call CreateLayer;
EndFor

////Important tags////
WallTag = 1514; //infinite boundary surface tag, number was lifted from GMSH interactive GUI and chagnes depending
on the number of surface elements defined

////////////////////////////////////
//subtracting magnet volume from vacuum volume//
////////////////////////////////////
vacVol = newv;
BooleanDifference(vacVol+0) = { Volume{bounds+0}; Delete; }{ Volume{Magnet[]}; };
MeshSize{ PointsOf{ Surface{WallTag}; } } = h; //Surface should be the infinite boundary

//Creating embeddeed points in the vacuum mesh along the probing region (z-axis)
p = newp; //embedded probing points in the center region
For i In {0:hP-1} //runs hP times
    Point(p+i) = {0,0, 2*i*(R/4)/hP -R/4 ,hM};
EndFor
Point{p+0:p+hP-1} In Volume{vacVol+0};

////////////////////////////////////
//Physical volumes//
////////////////////////////////////

Wall[0] = WallTag; //important for setting A=0 on boundary for PDE
BIASVOLUME = 10;
CURVVOLUME = 11;
VACUUMVOLUME = 20;
WALL = 1;

Physical Volume("Bias Coil", BIASVOLUME) = {Bias[]};
Physical Volume("Curv Coil", CURVVOLUME) = {Curv[]};
Physical Volume("Vacuum", VACUUMVOLUME) = {vacVol+0};
Physical Surface("Wall", WALL) = {Wall[]};

Save "~/GmshMeshes/BiasCurvatureV8";

```

Appendix B: Below is the sample code used for FreeFEM++ simulation,

```
//Implementation of FreeFEM solving the weak formulation of 3D magnetostatics given some meshed volume with current.
//Code is adapted from FreeFEM++'s modules documentation.
//This code is meant to operate with the mesh file generated from the GMSH code above.

include "ffmatlib-Edited.idp"
include "getARGV.idp"
load "gmsH"
load "iovtk"
load "medit"
load "PETSc" //PETSc plug in
load "MPICG"
load "MUMPS_FreeFem" // plugs to allow for more efficient memory usage for large meshes

//file iterable for use in conjunction with shell code to iterate over different geometries.
real fileNumber = getARGV("-file",1);

//Parameters
real Mu0 = 4.*pi*1.e-7; //Vacuum magnetic permeability
real MuC = 1.25e-6; //Copper magnetic permeability
real J0B = -200.e4; //Current density bias
real J0C = -275.e4; //Current density curvature

//Mesh parameters, needs to be the same as those used to generate Disc.msh
real R = 0.1200;
real H = 0.001;
real RCint = 0.0220;
real RCext = 0.0330;
real RBint = 0.0560;
real RBext = 0.0720;
real currentDirection = -1;
//1 means same direction, -1 means opposite directions.
//opposite directions means Curvature has clockwise current (looking from above),
//then bias has counter-clockwise current.

cout << "\n\nUsing file: BiasCurvatureV8-" + fileNumber << "\n\n";
mesh3 Th = gmshload3("GmshMesh/BiasCurvatureV8-" + fileNumber);

int BIASVOLUME = 10;
int CURVVOLUME = 11;
int VACUUMVOLUME = 20;
int BoxWall = 1; // Make sure this is the same as defined in the .geo file.

//FESpaces
func Pk = P1;
fespace Ah(Th, [Pk, Pk, Pk]);
Ah [Ax, Ay, Az] = [0, 0, 0];

//Macro
macro Curl(Ax, Ay, Az) [dy(Az)-dz(Ay), dz(Ax)-dx(Az), dx(Ay)-dy(Ax)] //
macro Divergence(Ax, Ay, Az) (dx(Ax) + dy(Ay) + dz(Az)) //

//Current
func r = sqrt(x^2+y^2);
Ah [Jx, Jy, Jz] = [
  ( J0B * cos(atan2(y,x)+pi/2.) * (Th(x, y, z-0.001).region==BIASVOLUME) ) +
  ( J0C * cos(atan2(y,x)+pi/2.) * (Th(x,y,z-0.001).region==CURVVOLUME) ) * currentDirection,
  ( J0B * sin(atan2(y,x)+pi/2.) * (Th(x,y,z-0.001).region==BIASVOLUME) ) +
  ( J0C * sin(atan2(y,x)+pi/2.) * (Th(x,y,z-0.001).region==CURVVOLUME) ) * currentDirection,
  0
];

cout << "Done Initializing... \n";

//Problem
func Mu = Mu0 + (MuC-Mu0)*(1*(Th(x,y,z-0.001).region==BIASVOLUME) +
  1*(Th(x,y,z-0.001).region==CURVVOLUME));
func Nu = 1. / Mu;

cout << "Finding Laplacian... \n";
varf vLaplacian ([Ax, Ay, Az], [AAx, AAy, AAz])
```

```

= int3d(Th)(
    Nu * Curl(Ax, Ay, Az)' * Curl(AAx, AAy, AAz)
    + (1./Mu) * Divergence(Ax, Ay, Az) * Divergence(AAx, AAy, AAz)
)
- int3d(Th)(
    [Jx, Jy, Jz]' * [AAx, AAy, AAz]
)
+ on(BoxWall, Ax=0, Ay=0, Az=0)
;

matrix<real> Laplacian = vLaplacian(Ah, Ah, solver=sparse solver, datafilename="ffmumps_fileparam.txt");
cout << "Done Finding Laplacian with size: " << Laplacian.n << "x" << Laplacian.m << "...\\n";
real[int] LaplacianBoundary = vLaplacian(0, Ah);
cout << "Done Finding Boundary... \\n";
Ax[] = Laplacian^-1 * LaplacianBoundary; //memory issue with inverting this matrix

//Magnetic induction
Ah [Bx, By, Bz];
[Bx, By, Bz] = -1* Curl(Ax, Ay, Az); //negative sign is because initial imulations predicted wrong directions for B vectors

//Magnetic field
Ah [Hx, Hy, Hz];
[Hx, Hy, Hz] = Nu * [Bx, By, Bz];

//VTU file format is for visualization in paraview.
int[int] ord = [1, 1, 1, 1];
savevtk("Results/BiasCurvatureV8_Z-" + fileNumber + ".vtu", Th, [Jx, Jy, Jz], [Bx, By, Bz], [Hx, Hy, Hz], [Ax, Ay,
    Az], dataname = "j B H A", order = ord, bin = 1);

cout << "\\n Done saving BiasCurvatureV8_Z-" + fileNumber + ".vtu";

```

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